No.
Date

# Unit - III

## Electromagnetic theory

**Del operator :-**

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} = \text{Nabla}$$

**Laplacian operator :-**

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2}{\partial \theta^2}\right) + \frac{\partial^2}{\partial z^2}$$

**Gradient of Scaler field :-**

$$dV = \vec{\nabla} V \cdot dr$$

where $\vec{\nabla} V$ is called gradient of V as grad V.

**Divergence of Vector field :-**

$$\text{div}\,\vec{A} = \vec{\nabla}\cdot\vec{A} = \iint_S \vec{A}\cdot\vec{ds} = \lim_{V\to 0} \frac{1}{V}\iint_S \vec{A}\cdot\vec{ds}$$

$$\text{div}\,\vec{E} = \lim_{V\to 0} \frac{1}{V}\iint_S \vec{E}\cdot\vec{ds} = \frac{\rho}{\varepsilon_0}$$

Curl of Vector field :-

$$\text{Curl } \vec{A} = \vec{\nabla} \times \vec{A}$$

If $\vec{\nabla} \times \vec{A} = 0$ then field is called irrotational

Gauss Divergence theorem :-

This theorem is used to transform a volume integral into surface integral.

Statement :- The surface integral of a vector field $\vec{A}$ over the closed surface S is equal to the volume integral of a divergence of the vector field $\vec{A}$ over the volume enclosed by the surface.

$$\boxed{\oiint_S \vec{A} \cdot \vec{ds} = \iiint_V \vec{\nabla} \cdot \vec{A} \, dV}$$

Stokes theorem :-

This theorem relates line integral to a surface integral.

Statement :-

The line integral to a vector field $\vec{A}$ around any closed curve C is equal to the surface integral of the curl $\vec{A}$ over an open surface S bounded by curve C.

$$\boxed{\oint \vec{A} \cdot \vec{dl} = \iint_S \vec{\nabla} \times \vec{A} \cdot \vec{ds}}$$

No

Date

## Electrodynamics before maxwell :-

(i) Gauss law in electrostatics -

$$\text{div}\,\vec{E} = \rho$$

or

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \quad \text{or} = \rho$$

(ii) Gauss law in magnetostatics -

$$\text{div}\,B = 0$$

or

$$\vec{\nabla}\cdot\vec{B} = 0$$

(iii) Faraday's law and lenz law of electromagnetic induction -

$$\text{Curl}\,\vec{E} = -\frac{\partial B}{\partial t} \quad \text{or} \quad \vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$$

(iv) Ampere's law :-

$$\text{Curl}\,\vec{B} = \mu_0 J \quad \text{or} \quad \vec{\nabla}\times\vec{B} = \mu_0 J$$

where $\mu_0$ = permeability of free space and $J$ = Current density

## Equation of Continuity :-

The law of conservation of charge can be expressed by the equation of continuity.

We know that the rate of flow of charge is called electric current.

$$I = -\frac{dq}{dt} \quad \text{——— (1)}$$

The charge density at the point of a conductor is $\rho$ and the charge with in volume $dV$ about that point is $dq$, then -

$$\rho = \frac{dq}{dV}$$

$$dq = \rho\, dV$$

$$q = \int_V \rho\, dV$$

from equⁿ (1)

$$I = -\frac{d}{dt}\int_V \rho\, dV \quad \text{——— (2)}$$

Now the current through whole of the surface S

$$I = \int_s J \cdot ds \quad \text{——— (3)}$$

from equⁿ (2) and (3)

$$\int_s J \cdot ds = -\frac{d}{dt}\int_V \rho\, dV \quad \text{——— (4)}$$

using gauss divergence theorem in L.H.S we get

$$\int_s \bar{J} \cdot \bar{ds} = \int_V (\nabla \cdot J)\, dV$$

So equⁿ (4) becomes -

$$\int_V (\nabla \cdot J)\, dV = -\int_V \frac{d}{dt} \rho\, dV$$

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

$$\boxed{\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0}$$

This equ$^n$ is called equ$^n$ of continuity.

For steady state, the charge density to any point is constant

$$\frac{\partial \rho}{\partial t} = 0$$

hence $\boxed{\nabla \cdot J = 0}$

Modification of Ampere's law :-

We know that the differential form of Gauss law is

$$\vec{\nabla} \cdot \vec{E} = \rho / \varepsilon_0$$

$$\rho = \varepsilon_0 \vec{\nabla} \cdot \vec{E}$$

differentiating both the side w.r.to t, we get -

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t} (\varepsilon_0 \vec{\nabla} \cdot \vec{E})$$

$$\frac{\partial \rho}{\partial t} = \varepsilon_0 \nabla \cdot \frac{\partial E}{\partial t}$$

adding $\nabla \cdot J$ both the side, we get

Scanned with CamScanner

$$\bar{\nabla}\cdot\bar{J} + \frac{\partial \rho}{\partial t} = \bar{\nabla}\cdot\bar{J} + \epsilon_0 \nabla\cdot\frac{\partial \bar{E}}{\partial t}$$

$\bar{\nabla}\cdot\bar{J} + \frac{\partial \rho}{\partial t} = 0$ is the equ$^n$ of continuity for varrying current.

So we get -

$$\bar{\nabla}\cdot\bar{J} + \epsilon_0 \nabla\cdot\frac{\partial \bar{E}}{\partial t} = 0$$

$$\boxed{\nabla\cdot\left[\bar{J} + \epsilon_0\frac{\partial \bar{E}}{\partial t}\right] = 0}$$

This is another equ$^n$ of continuity for varrying current.

Maxwell therefore proposed that for varrying current J in ampere's law, replaced J by $J + \epsilon_0\frac{\partial E}{\partial t}$. So we get

$$\boxed{\text{Curl}\,\bar{B} = \bar{\nabla}\times\bar{B} = \mu_0\left[J + \epsilon_0\frac{\partial E}{\partial t}\right]}$$

For steady current, the term $\epsilon_0\frac{\partial E}{\partial t}$ becomes zero.

In empty space where there is no current

$$\bar{J} = 0$$

$$\boxed{\bar{\nabla}\times B = \mu_0\epsilon_0\frac{\partial \bar{E}}{\partial t}}$$

$\frac{\partial E}{\partial t}$ is the rate at which electric current is changing between the plate of the capacitor.

Equ$^n$ $\nabla \times \bar{B} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$ is a varrying electric field between the plate of the capacitor gives rise to a magnetic field.

So that the varrying electric field is another source of magnetic field.

Types Of Current

- True Current
- Other Current
  - Polarization Current
  - Magnetization Current
  - Convective Current
  - Displacement Current