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Elements of Modern Physics.
Paper II

Zeeman Effect and
Paschen Back Effect

N zeeman Co, Cu, Zn, Cd etc

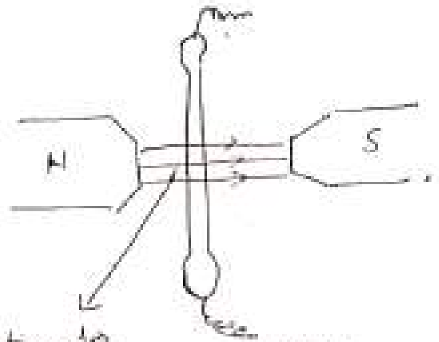
Zeeman Effect :- Zeeman found in 1896 that when sodium light is passed through the space between the pole-pieces of a powerful magnet the two D-lines are considerably broadened. This is known as Zeeman effect. Preston later showed, using instrument having greater dispersion and resolving power, that certain lines were split into as many as four and even six components. The zeeman effect is a clear confirmation of space quantisation. It is a magneto-optical phenomenon.

— magnetic field more than 2 Tesla

The Zeeman effect is of two types (a) the normal zeeman effect (b) the anomalous Zeeman effect.

When a spectral line splits into two or three lines, it is called "Normal Zeeman effect" and when a spectral line splits it into more than three lines in presence of ordinary magnetic field, it is called "anomalous zeeman effect".

Normal Zeeman effect can be explained by classical theory, considering only the orbital motion of e^- whereas anomalous zeeman effect can't be explained by classical theory.

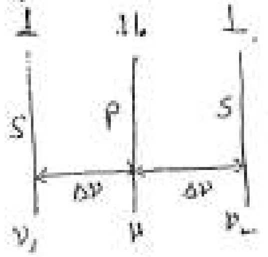


Normal Zeeman effect (experimental arrangement)

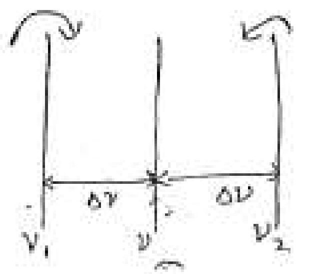
Experimental arrangement for Zeeman effect :- Experimental arrangement is shown in figure. Here source of light giving line spectrum is put in a magnetic field produced by an electro-magnet having conical pole pieces with holes drilled across them. When spectral line is viewed with a high resolving power microscope, more than one lines are observed.

It may be observed in two ways. In the first one, the source of light is placed in a strong magnetic field and observations on the lines emitted by it are made in a direction perpendicular to magnetic field direction. This is known as transverse Zeeman effect. In the second arrangement the light emerging in a direction parallel to the field lines is examined, this is known as longitudinal zeeman effect.

In the case of normal transverse Zeeman effect, a single line of frequency ν in the absence of magnetic field splits into three lines of frequency ν_1, ν, and ν_2 when the magnetic field is applied and the lines of frequencies ν_1 and ν_2 are symmetrically situated on either side of the position of the original line of frequency ν.



normal transverse zeeman effect



normal longitudinal zeeman effect

In the case of longitudinal Zeeman effect, the lines of original frequency disappears and two new lines of frequencies ν_1 and ν_2 appear when field is applied. The light in these splitted lines is polarised. In the case of normal transverse Zeeman effect, the electric vector of light in the line of unaltered frequency ν vibrates in a direction parallel to the direction of applied magnetic field whereas the electric vectors of light of frequencies ν_1 and ν_2 are polarised in a direction $\perp$ to the magnetic field direction and the direction of observation. The symbol p stands for vibrations parallel to the field and s stands for vibration normal to the field. In the case of normal longitudinal Zeeman effect the lines of frequencies ν_1 and ν_2 consists of circularly polarised light, shown by arrows in figure (5), with the direction of one being opposite to that of the other.

Imp The normal Zeeman effect occurs only in the case of atoms which have an even number of electrons and only if the electron spins mutually compensate each other.

The wavelength separation between lines is called Z-shift and is given by

$$\boxed{\delta\lambda = \frac{eB\lambda^2}{4\pi mc}}$$

$$\Delta\lambda = \delta\lambda = \frac{eB\lambda^2}{4\pi mc}$$

Calculation of wavelength shift ($\delta\lambda$):- Suppose an electron is moving in a circular orbit of radius r with a linear velocity v and angular velocity ω. In the absence of magnetic field, centripetal force on electron is given by,

$$F = \frac{mv^2}{r} = m\omega^2 r \quad \text{----(1)} \qquad \text{using, } v = r\omega$$

where m is mass of electron

Now, if an external magnetic field B is applied in a direction perpendicular to the plane of the orbit, a magnetic Lorentz force eVB acts outwards for clockwise motion of electron while it acts towards centre of orbit for anticlockwise motion of electron.

If $\delta\omega$ be the change in angular ~~frequency~~ velocity of electron, then for clockwise motion of electron,

$$F - eVB = m(\omega+\delta\omega)^2 r$$

$$m\omega^2 r - eVB = m(\omega+\delta\omega)^2 r$$

$$= m(\omega^2 + \delta\omega^2 + 2\omega\,\delta\omega) r \qquad \text{| neglecting } (\delta\omega)^2$$

$$= (m\omega^2 + 2m\omega\,\delta\omega) r$$

$$-eVB = 2m\omega\,\delta\omega \qquad \text{putting } V = r\omega$$

$$\boxed{\delta\omega = -\frac{eB}{2m}}$$

Similarly for anticlockwise motion of electron we can obtain

$$\delta\omega = \frac{eB}{2m}$$

Due to magnetic Lorentz force, the orbital angular momentum vector 'l' precesses around the direction of magnetic field and the resulting motion of electron is called LARMOR PRECESSION. The angular ~~frequency~~ velocity of Larmor Precession is given by,

$$\delta\omega = \pm \frac{eB}{2m}$$

If ν is the frequency, then

$$\omega = 2\pi\nu.$$

or $$\delta\omega = 2\pi\,\delta\nu.$$

or $$\delta\nu = \frac{1}{2\pi}\delta\omega.$$

$$= \pm \frac{eB}{4\pi m}.$$

$c = \nu\lambda$

or $\nu = \frac{c}{\lambda}$

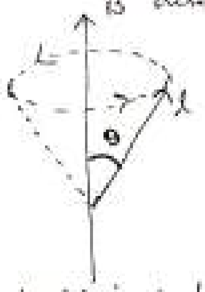
$\delta\nu = -\frac{c}{\lambda^2}\delta\lambda.$

In terms of wavelength, above expression is written as

$$-\frac{c}{\lambda^2}\delta\lambda = \pm \frac{eB}{4\pi m}$$

$$\boxed{\delta\lambda = \mp \frac{eB\lambda^2}{4\pi mc}}$$

This is the required expression.



Quantum Mechanical Explanation of normal Zeeman effect :- When magnetic field is not weak, we can neglect spin motion of electron.

The orbital angular momentum of an electron moving in an orbit is $l\frac{h}{2\pi}$. When a uniform magnetic field, B is applied, the orbit of the electron processes about an axis through the nucleus parallel to the magnetic field with a uniform frequency,

$$\nu_L = \frac{eB}{4\pi m}$$

The vector representing the orbital momentum of the electron will also process with the same frequency ν_L. This precession of the orbit will add to the kinetic energy of the electron. The angle between the vector $\vec{l}$ and the direction of magnetic field can assume only certain values given by,

$$\cos\theta = \frac{m_l}{l}$$

m_l

where m_l can assume $(2l+1)$ values, namely $l, l-1, l-2, \ldots 0, -1, -2, \ldots l$.
↓
magnetic orbital quantum number.

The orbital magnetic moment of the electron is,

$$\mu_l = -\frac{eh}{4\pi m}\vec{l}$$

The magnetic dipole moment μ_l of the orbiting electron will be directed opposite → to $\vec{l}$. The potential energy of the dipole inclined at angle θ with the field will be $-\vec{\mu_l}\cdot\vec{B}$. Therefore the additional energy in this position will be

$$\Delta E = -\mu_l \cdot \vec{B} = -\mu_l B \cos\theta$$

$$= \left(\frac{eh}{4\pi m}l\right)B\left(\frac{m_l}{l}\right) = m_l \frac{eh}{4\pi m}B$$

where m_l can have $(2l+1)$ values.

→ Thus, the effect of applied magnetic field is to split one energy level determined by l into $(2l+1)$ energy levels. The spacing between these splitted levels is the same for all levels. Therefore the number of possible transitions and also the number of lines in the spectrum will increase.

without magnetic field Let us consider, as an example, the case of an electron in an atom which undergoes an allowed transition $(\Delta l = \pm 1)$ from $l=2$ to $l=1$. This will result in the emission of a single line of frequency ν given by $\frac{E_2 - E_1}{h}$.

When magnetic field is applied the energy level corresponding to $l=1$ will split into $(2\times1+1)=3$ and that corresponding to $l=2$ will split into $(2\times2+1=5)$ 5 levels. Therefore there will be five values of E_{2B} and three values of E_{1B} where E_B is the energy of the electron in the presence of magnetic field and the spacing between the levels will be the same, i.e., $\frac{ehB}{4\pi m}$ for both $l=2$ and $l=1$.

The frequencies ν for the lines emitted under the influence of magnetic field will then be,

$$\nu_B = \frac{E_{2B} - E_{1B}}{h}$$

Now, $E_{1B} = E_1 + \Delta E$ or

$$E_{1B} = E_1 + m_{l_1}\frac{eh}{4\pi m}B$$

$$E_{2B} = E_2 + \Delta E$$

$$E_{2B} = E_2 + m_{l_2}\frac{eh}{4\pi m}B$$

Substituting these in above equation, we get

$$\nu_B = \frac{E_2 - E_1 + \Delta m_l \frac{eh}{4\pi m}B}{h}$$

→ where Δm_l can have values ±1 or 0. As a consequence of equal spacing between the levels all transitions with the same Δm_l give the same frequency.

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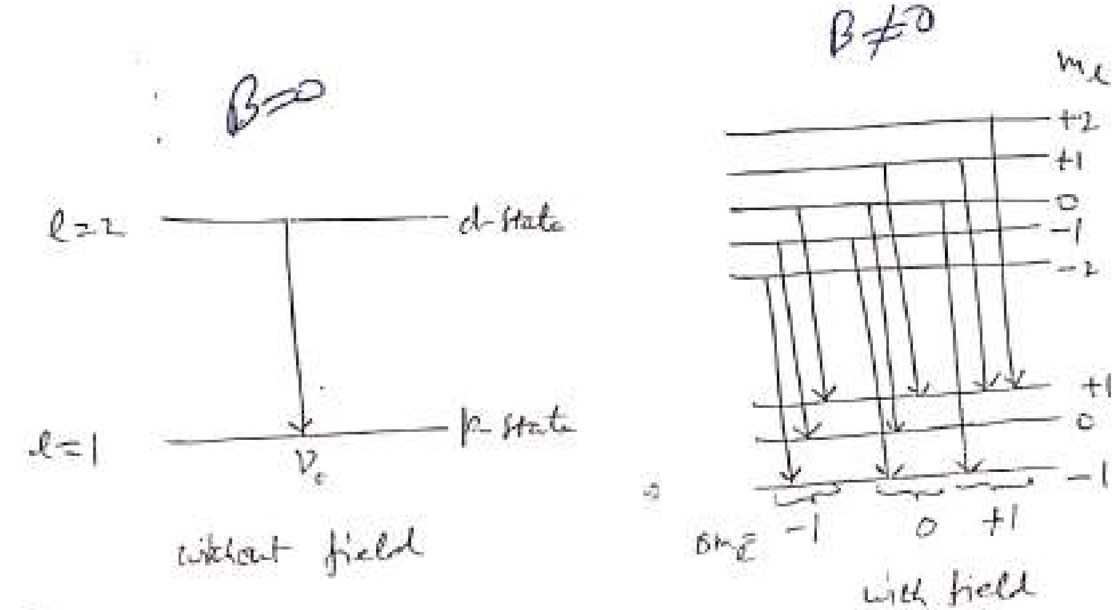
Considering selection rule $\Delta m_l = 0$ or ± 1. We can show that a single spectral line splits into three components.

In figure three transitions in a bracket indicate same spectral line



In wave numbers, the interaction energy is

$$-\Delta T = \frac{\Delta E}{hc} = \frac{eB}{4\pi mc} m_l \qquad \text{(from Eq. B)}$$

All transitions corresponding to same Δm_l coincide in wave number. This gives the normal triplet pattern, with three component lines.

The rule $\Delta m_l = 0$ gives the π^* component in the position of the field-free line while $\Delta m_l = \pm 1$ give the two symmetrically displaced σ components.

The wave number separation between consecutive components is equal to the separation between consecutive Zeeman levels, and is given by

$$\Delta\nu = L' = \frac{eB}{4\pi mc}$$

Imp — where L' is called the 'Lorentz unit',

putting $e = 1.6\times10^{-19}$ C, $m = 9.1\times10^{-31}$ kg, $c = 3\times10^{8}$ m/s, we get

$$\boxed{\Delta\nu = 46.7\, B\ m^{-1}}$$

where B is in tesla (N/A-m).

* For observation at right angles to the magnetic field, the transition $\Delta m_l = 0$ results in spectral line polarised with the electric vector parallel to the field (π component), while the transition $\Delta m_l = \pm 1$ result in lines polarised with the electric vector perpendicular to the field (σ component

<u>Anomalous Zeeman Effect :-</u> The spectral lines emitted by some elements split into four, six, or even more components instead of three under the influence of a magnetic field, it is further found that even if a spectral line splits into three components, their spacing may not agree with $\Delta\nu = \frac{eB}{4\pi m}$, this is known as anomalous Zeeman effect.

This can be explained by using the idea of spin of electron.

<u>Let us consider a one electron system</u> i.e, the case in which core of the atom and its nucleus has zero magnetic and mechanical moments and any moments of the atom are due to a single electron.

The electron then has two angular momentum vectors, namely orbital angular momentum l^* and spin momentum s^*. The total angular momentum

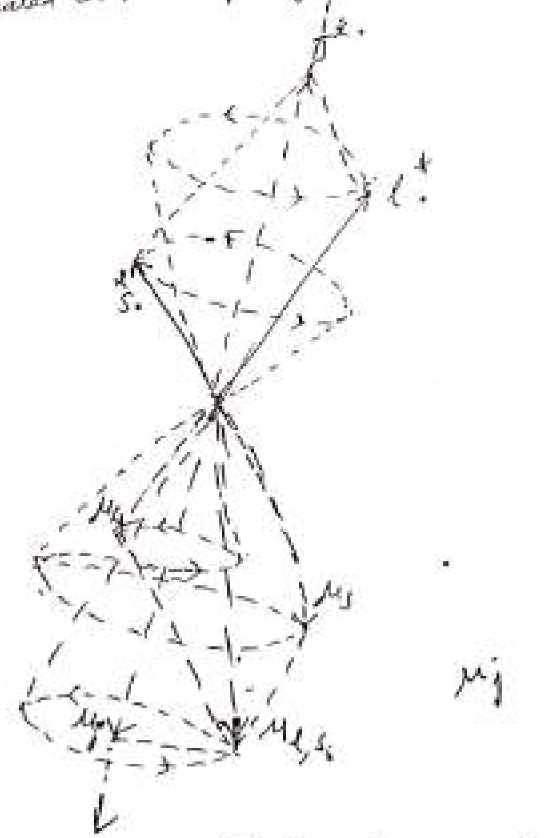
$$j^* = l^* + s^*$$

The magnetic moment associated with the orbital motion of the electron is $\mu_l = l^* \frac{eh}{4\pi m}$ and the magnetic moment associated with the spin of the electron is

$$\mu_s = 2s^* \cdot \frac{eh}{4\pi m}$$

μ_l will be directed opposite to l^* and μ_s opposite s^* due to the negative charge of e^-.

The ratio between the magnetic moment μ_l and mechanical moment $l\frac{h}{2\pi}$ of an e^- in orbit is $\frac{e}{2m}$ and the ratio for the spin of the electron is $\frac{2e}{2m}$.



Vector model showing the magnetic and mechanical moments of a single valence electron.

A Schematic diagram of the magnetic moments (μ_l, μ_j, μ_s) and mechanical moments (l^* and s^*) is shown in figure. Here it can be noted that the resultant magnetic moment $\mu_{l,s}$ is not in line with the mechanical moment $j^*\frac{h}{2\pi}$. As l^* and s^* precess about j^*, μ_l and μ_s must also precess around j^*. As a consequence of this precession the components of μ_l and μ_s perpendicular to j^* will average out to zero over one period of the electron and the components of μ_l and μ_s parallel to j^* will contribute to the magnetic moment of the atom.

(~~[illegible]~~)

Schematic diagram of magnetic and

258 A

The Components of μ_l and μ_s parallel to j^* are $l^* \frac{eh}{4\pi m} \cos(l^*, j^*)$ and $2s^* \frac{eh}{4\pi m}(s^*, j^*)$.
Adding these we get

$\boxed{\mu_l \cos(\mu_l, \mu_j)}$ $\boxed{\mu_s \cos(\mu_s, \mu_j)}$

$$\mu_j = \left[l^* \cos(l^*, j^*) + 2s^* \cos(s^*, j^*)\right] \frac{eh}{4\pi m}$$

According to cosine law

$$\cos(l^*, j^*) = \frac{l^{*2} + j^{*2} - s^{*2}}{2 l^* j^*}$$

$$\cos(s^*, j^*) = \frac{s^{*2} + j^{*2} - l^{*2}}{2 s^* j^*}$$

Substituting in above equation, we get

$$\mu_j = \left[\frac{l^{*2} + j^{*2} - s^{*2}}{2 j^*} + 2\, \frac{s^{*2} + j^{*2} - l^{*2}}{2 j^*}\right] \frac{eh}{4\pi m}$$

$$= \frac{eh}{4\pi m}\left[\frac{3j^{*2} + s^{*2} - l^{*2}}{2 j^*}\right]$$

$$= \frac{eh}{4\pi m} j^* \left[1 + \frac{j^{*2} + s^{*2} - l^{*2}}{2 j^{*2}}\right]$$

Now $l^* = \sqrt{l(l+1)}$, $s^* = \sqrt{s(s+1)}$ and $j^* = \sqrt{j(j+1)}$

$$\therefore \quad \mu_j = \frac{eh}{4\pi m} j^* \left[1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2 j (j+1)}\right]$$

The quantity within the brackets in the above expression is denoted by 'g' and is known as "Lande g-factor";

Therefore

$$\mu_j = \frac{eh}{4\pi m} g j^*$$

where

$$g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2 j (j+1)}$$

Imp If the atom is placed in a magnetic field, torque are exerted on both l^* and s^* causing the vector j^* to precess around the magnetic field. Due to anomalous spin magnetic moment of the electron, s^* tends to precess around $\vec{B}$ twice as fast as is done by the orbital momentum vector l^*. If the magnetic field $\vec{B}$ is not too strong, the coupling between l^* and s^* is sufficiently strong to maintain a constant j^* which will precess around the direction of magnetic field with the angular velocity given by Larmor theorem

$$\omega_L = \frac{eB}{2m} g$$

The additional energy ΔE of the atom due to precession around the direction of magnetic field is given by the precessional angular velocity ω_L times the component of the resultant mechanical moment $j^* \frac{h}{2\pi}$ on the axis of rotation B.

$$\Delta E = \omega_L j^* \frac{h}{2\pi} \cos(j^*, \vec{B}) \qquad \omega_L = \frac{eB}{2m} g$$

$$= \frac{eh}{4\pi m} g \, j^* B \cos(j^*, \vec{B})$$

But $j^* \cos(j^*, \vec{B}) = m_j \rightarrow$ the projection of vector j^* on the direction of magnetic field called magnetic quantum number, therefore

$$\Delta E = \frac{eh}{4\pi m} g B m_j$$

Dividing ΔE by hc, the interaction energy in wave number becomes

$$\frac{\Delta E}{hc} = -\Delta T = \frac{eB}{4\pi mc} g m_j$$

Since the field B is the same for all energy levels of a given atom it is convenient to express the Zeeman splitting in terms of the quantity $\frac{eB}{4\pi mc}$ referred to as Lorentz unit and denoted by L. Then

$$-\Delta T = g m_j L \qquad \text{(magnetic shift)} \qquad L = \frac{eB}{4\pi mc}$$

Here ΔT is the change in energy for each m_j level from original level and the shift is proportional to the field strength B.

Notation of the levels

Level	l	s	multiplicity $(2s+1)$	$j = l \pm s$	Spectral Notation $^{(2s+1)}\text{Level}_j$
S	0	$\frac{1}{2}$	2	$\frac{1}{2}$	$^2S_{\frac{1}{2}}$
P	1	$\frac{1}{2}$	2	$\frac{3}{2}, \frac{1}{2}$	$^2P_{\frac{3}{2}}, \; ^2P_{\frac{1}{2}}$
D	2	$\frac{1}{2}$	2	$\frac{5}{2}, \frac{3}{2}$	$^2D_{5/2}, \; ^2D_{3/2}$
F	3	$\frac{1}{2}$	2	$\frac{7}{2}, \frac{5}{2}$	$^2F_{\frac{7}{2}}, \; ^2F_{\frac{5}{2}}$

The transitions between these split-levels are governed by the selection rule

$$\Delta m_j = 0, \pm 1$$

They give rise to the magnetic splitting of the spectral line. The number of components into which a line splits up is in general different from that expected (two or three) when the electron spin is not considered (normal Zeeman effect). Hence it is known as the anomalous Zeeman effect.

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Let us consider the splitting of sodium principal series doublet D-lines under the influence of magnetic field.

Intensity rule

$\Delta L = -1, \Delta J = -1 \rightarrow$ Strongest

$\Delta L = -1, \Delta J = 0 \rightarrow$ less intense

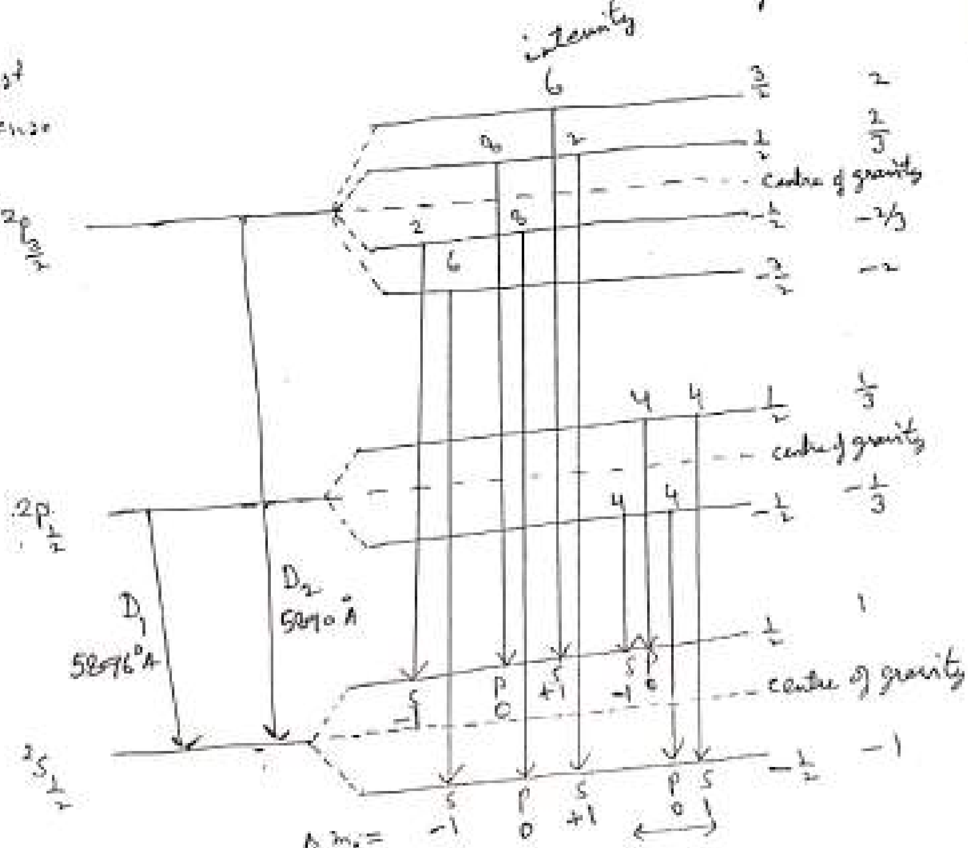
$\Delta L = -1, \Delta J = +1 \rightarrow$ weak

$\Delta L = +1, \Delta J = 0 \rightarrow$ very weak

$\Delta L = -1, \Delta J = +1 \rightarrow$ forbidden

$\Delta L = +1, \Delta J = +1 \rightarrow$ forbidden

m_j has $(2j+1)$ values.



Splitting of D-lines of sodium under the influence of magnetic field.

Since m_j has $(2j+1)$ values, any energy level will split into $(2j+1)$ sub-levels when magnetic field is applied as shown in above figure and the spacing between the levels will be proportional to $g m_j$. Applying selection rule that m_j can only change such that $\boxed{\Delta m_j = 0 \text{ or } \pm 1}$ only the transitions between the various energy levels for sodium D-lines ~~are possible~~ are shown in above figure.

- In the case of the ground level $^2S_{\frac{1}{2}}$, $l=0$, $s=\frac{1}{2}$, $j=\frac{1}{2}$, therefore m_j will have $2 \times \frac{1}{2} + 1 = 2$ values i.e. $\frac{1}{2}$ and $-\frac{1}{2}$ and $g = 1 + \frac{\frac{1}{2}(\frac{1}{2}+1) + \frac{1}{2}(\frac{1}{2}+1)}{2 \times \frac{1}{2}(\frac{1}{2}+1)} = 2$ | $g = 1 + \frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}$

Therefore $g m_j$ can have two values namely $+1$ and -1.

- Similarly $^2P_{\frac{1}{2}}$ splits into two levels and $^2P_{\frac{3}{2}}$ level splits into four levels according to the values of 'g' and 'm_j' for each energy level given in Table.
- The dotted lines in the figure represents the centres of gravity of the associated levels.

Extra

Notation $\rightarrow$ $n\,{}^{2}S_{\frac{1}{2}}$

multiplicity $(2S+1)$

level $(S, P, D, F, \ldots)$

number of the orbit

Value of J, $(J = L \pm S)$

$2\,{}^{2}P_{\frac{1}{2}}$

possible value of J lies between $L-S$ to $L+S$

for p state $L=1$, $S=\frac{1}{2}$

J values $1-\frac{1}{2}$ to $1+\frac{1}{2}$

Table $m_j = -j \dots j$ Values $g m_j = \beta_{?} (j^2 + ?)$

State	l	s	j	g	m_j	$g m_j$
$(2s+1) = {}^2S_{\frac{1}{2}}$	0	$\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{1}{2}, -\frac{1}{2}$	$1, -1$
${}^2P_{\frac{1}{2}}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$2/3$	$\frac{1}{2}, -\frac{1}{2}$	$\frac{1}{3}, -\frac{1}{3}$
${}^2P_{\frac{3}{2}}$	1	$\frac{1}{2}$	$\frac{3}{2}$	$4/3$	$\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$	$2, \frac{2}{3}, -\frac{2}{3}, -2$

As shown in figure, the D_1 line which arises due to the transition ${}^2P_{\frac{1}{2}} \to {}^2S_{\frac{1}{2}}$ splits into four lines and D_2 line which results due to the transition ${}^2P_{\frac{3}{2}} \to {}^2S_{\frac{1}{2}}$ splits into six lines.

The same result has been found experimentally.

Intensity rule :- The intensity rules first derived by Burger, Dorgelo and Ornstein can be better expressed in terms of formulas abbreviated as follow:

$$\text{Transition } j \to j \begin{cases} m_j \to m_j \pm 1, & I = A(j \pm m_j + 1)(j \pm m_j) \\ m_j \to m_j, & I = 4A m_j^2 \end{cases}$$

$$\text{Transition } j \to j+1 \begin{cases} m_j \to m_j \pm 1 ; & I = B(j \pm m_j + 1)(j \pm m_j + 2) \\ m_j \to m_j ; & I = 4B(j + m_j + 1)(j - m_j + 1) \end{cases}$$

Here A and B are constant that need not be determined for relative intensities within a given Zeeman pattern.

Transitions and energy levels of Sodium D-lines in weak magnetic field

Sodium atom have atomic no. = 11, means 2, 8, 1

Spectral line are $1s^2, 2s^2, 2p^6, 3s^1, p$

Here transition is due to upper 'p' state to the 's' state

For outer most e^{-1}, 'p'

upper state $L=1$, $J=\frac{3}{2}, \frac{1}{2}$

Lower s state $L=0$, $J=\frac{1}{2}$

$J = L \pm S$
$J = L \pm \frac{1}{2}$
$= 1+\frac{1}{2} \rightarrow 1-\frac{1}{2}$
$= \frac{3}{2}, \frac{1}{2}$

m_j has $(2j+1)$ values, so any given level breaks up into $(2j+1)$ ~~Sub~~ sublevels.
Selection rule is $\Delta m_j = 0$ or ± 1.

For state $^{2}S_{\frac{1}{2}}$ (M_s, L, J), Multiplicity $2S+1 = 2 \Rightarrow S=\frac{1}{2}$

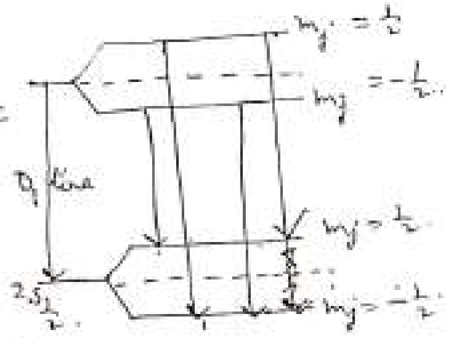
$J=\frac{1}{2}$

For $S \Rightarrow L=0$

$l=0$, $s=\frac{1}{2}$, $j=\frac{1}{2}$

so $$g = 1 + \frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}$$

$$= 1 + \frac{\frac{1}{2}(\frac{1}{2}+1)+\frac{1}{2}(\frac{1}{2}+1)-0}{2\frac{1}{2}(\frac{1}{2}+1)} = 1+1 = 2$$

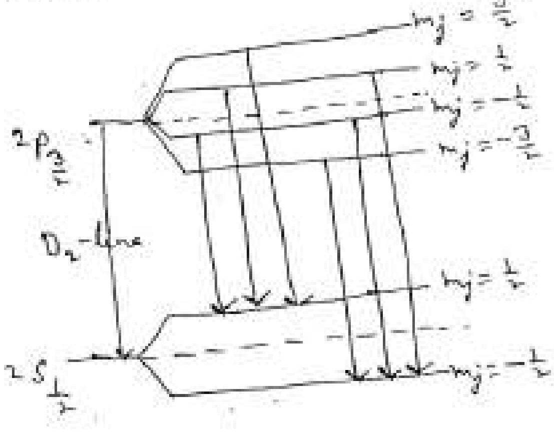


Thus, state $^2S_{\frac{1}{2}}$ splits up into two sublevels.
Similarly state $^2P_{\frac{1}{2}}$ splits into Imp $\{ m_j = 2j+1 = 2\cdot\frac{1}{2}+1 = 2$ so split into 2 sublevels $\}$

$l=1$, $2S+1=2 \Rightarrow S=\frac{1}{2}$, $J=\frac{1}{2}$

$$g = 1 + \frac{\frac{1}{2}(\frac{1}{2}+1)+\frac{1}{2}(\frac{1}{2}+1)-(1+1)}{2\cdot\frac{1}{2}(\frac{1}{2}+1)}$$

$$= 1 + \frac{\frac{3}{2}-2}{3/2} = 1 + \frac{-\frac{1}{2}}{3/2} = 1-\frac{1}{3} = \frac{2}{3}$$



State $^2P_{\frac{1}{2}}$ splits into two sublevels and State $^2P_{\frac{3}{2}}$ splits up into four sublevels.

Then D_1 line $(^2P_{\frac{1}{2}} \rightarrow {}^2S_{\frac{1}{2}})$ splits into 4 lines and D_2 line $(^2P_{\frac{3}{2}} \rightarrow {}^2S_{\frac{1}{2}})$ splits into six lines.

Imp $\{ m_j = 2j+1 = 2\cdot\frac{3}{2}+1 = 4$ so levels split into 4 sublevels

$^2P_{\frac{3}{2}}$, $2S+1=2 \Rightarrow S=\frac{1}{2}$

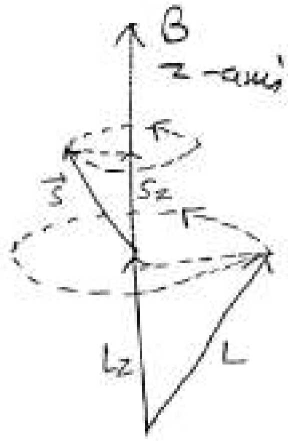
$P \rightarrow L=1$, $J=\frac{3}{2}$

$$g = 1 + \frac{\frac{3}{2}(\frac{3}{2}+1)+\frac{1}{2}(\frac{1}{2}+1)-(1+1)}{2\cdot\frac{3}{2}(\frac{3}{2}+1)}$$

$$= 1 + \frac{\frac{3}{2}\cdot\frac{5}{2}+\frac{1}{2}\cdot\frac{3}{2}-2}{3\cdot\frac{5}{2}} = 1 + \frac{\frac{15+3}{4}-2}{\frac{15}{2}} = 1 + \frac{\frac{9-4}{4}}{15/2} = 1 + \frac{\frac{5}{4}}{15/2} = 1 + \frac{1}{3\cdot 2}$$
$$= 1 + \frac{1}{6} = \frac{7}{6}$$

Paschen-Back Effect :- In the zeeman effect, the external magnetic field is weak as compared with the internal fields due to the spin and the orbital motion of the valence electron. When, however, the strength of the external field is increased, the separation between zeeman components increase until they become greater than the separation between multiplet fine-structure components. The anomalous zeeman pattern then changes over to like a normal zeeman pattern. This phenomenon is known as "Paschen-Back effect". It can be explained in the following way:

When the external field $\vec{B}$ becomes stronger as compared with the internal fields, the magnetic coupling between $\vec{J}$ and $\vec{B}$ exceeds that between $\vec{L}$ and $\vec{S}$, and the precession of $\vec{J}$ about $\vec{B}$ becomes faster than that of $\vec{L}$ and $\vec{S}$ about $\vec{J}$. Under these conditions the coupling between $\vec{L}$ and $\vec{S}$ is partially broken down and $\vec{J}$ is no longer fixed in magnitude. As the field $\vec{B}$ is further increased $\vec{L}$ and $\vec{S}$ start precessing independently about $\vec{B}$ with quantized components L_z and S_z along the field direction (z-axis), as shown in figure.



The magnitudes of these components are $m_L \frac{h}{2\pi}$ and $m_S \frac{h}{2\pi}$ respectively, where the magnetic quantum number m_L and m_S take the following discrete values:

$$m_L = L, L-1, L-2, \ldots, -L$$

and

$$m_S = S, S-1, S-2, \ldots, -S$$

By Larmor's theorem, the angular velocities of precession of $\vec{L}$ and $\vec{S}$ about the field $\vec{B}$ are given by the product of B and the corresponding ratios of the magnetic moments with the angular momenta. That is,

$$\omega_L = \frac{e}{2m} B$$

and

$$\omega_S = 2 \frac{e}{2m} B$$

The energy of each precession is equal to the product of the angular velocity and the projection of the corresponding angular momentum vector along the field direction. That is

$$\Delta E_L = \omega_L L_z = \frac{e}{2m} B m_L \frac{h}{2\pi}$$

and $$\Delta E_S = \omega_S S_z = 2\frac{e}{2m} B m_S \frac{h}{2\pi}$$

The sum of these two interaction energies is the main energy shift ΔE from the unperturbed energy level. Thus

$$\Delta E = \Delta E_L + \Delta E_S$$
$$= (m_L + 2m_S)\frac{eh}{4\pi m} B$$

The shift in wave number is

$$-\Delta T = \frac{\Delta E}{hc} = (m_L + 2m_S)\frac{eB}{4\pi mc}$$

or in Lorentz unit $\frac{eB}{4\pi mc} = L$

$$-\Delta T = (m_L + 2m_S) L$$

This is the expression for the strong-field magnetic interaction energy, ignoring spin-orbit interaction at all.

It shows that each of the field-free level is split into $(2L+1)(2S+1)$ magnetic levels.

In a strong field, the selection rules for transitions are

$\Delta m_L = 0$ (Components polarised $\parallel$ to the field)

$\Delta m_L = \pm 1$ (Components polarised $\perp$ to the field)

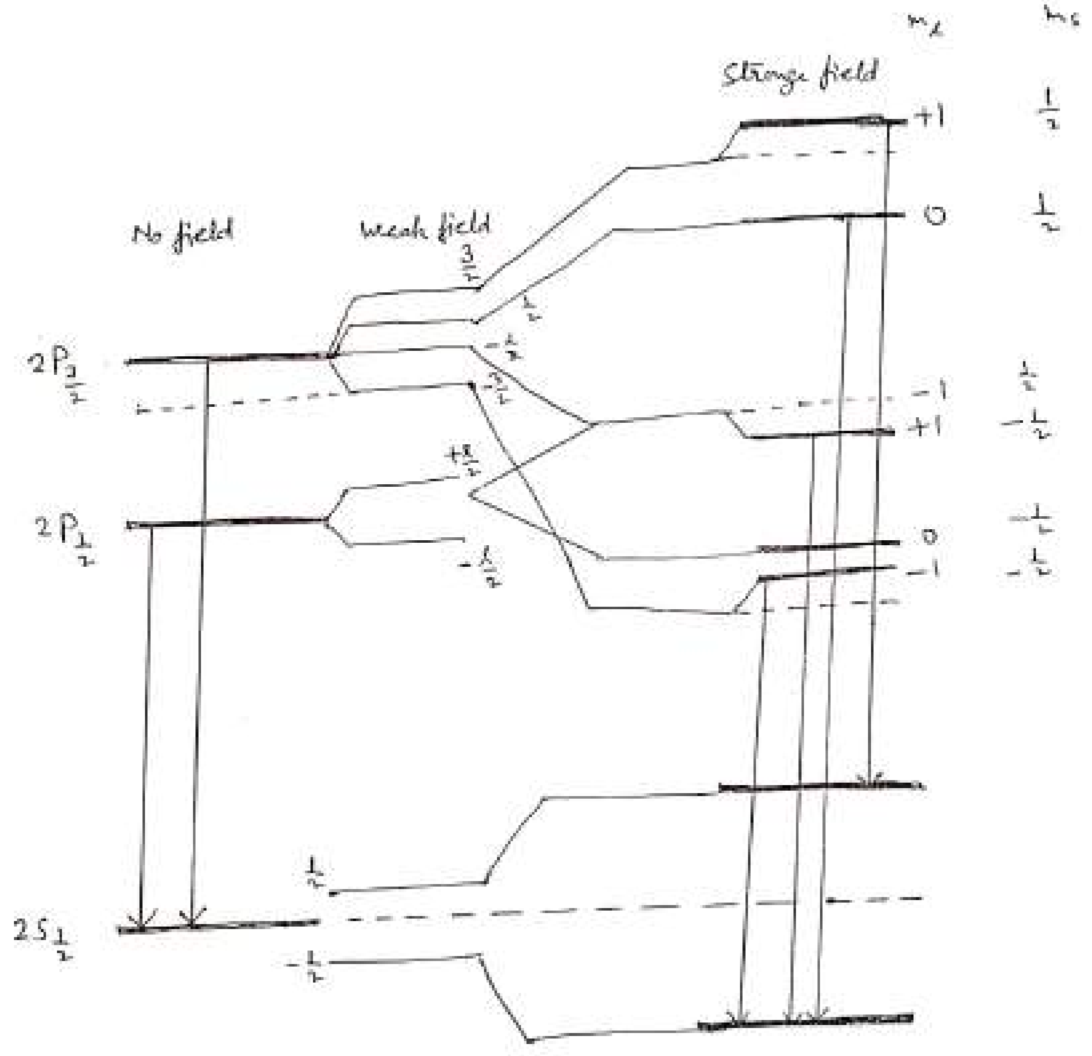
$\Delta m_S = 0$

When these selection rules are applied - they lead to a pattern same as a normal zeeman triplet.

Energy levels for a principal series doublet $^2S_{\frac{1}{2}} - {^2P_{\frac{1}{2}}}$ for both cases i.e. Zeeman and Paschen-Back effect.

~~Selection rule~~ :- Selection rule for Paschen-Back effect is $\Delta m_l = 0, \pm 1$ and $\Delta m_s = 0$.

As an example of the Paschen-Back effect the energy levels for the principal series doublets $2S_{\frac{1}{2}} - {^2P_{\frac{1}{2}, \frac{3}{2}}}$ are shown in figure.



Energy levels for a principal series doublet starting with no field at the left, weak field (Zeeman effect) in the middle and with a strong field (Paschen-Back effect) at the right.

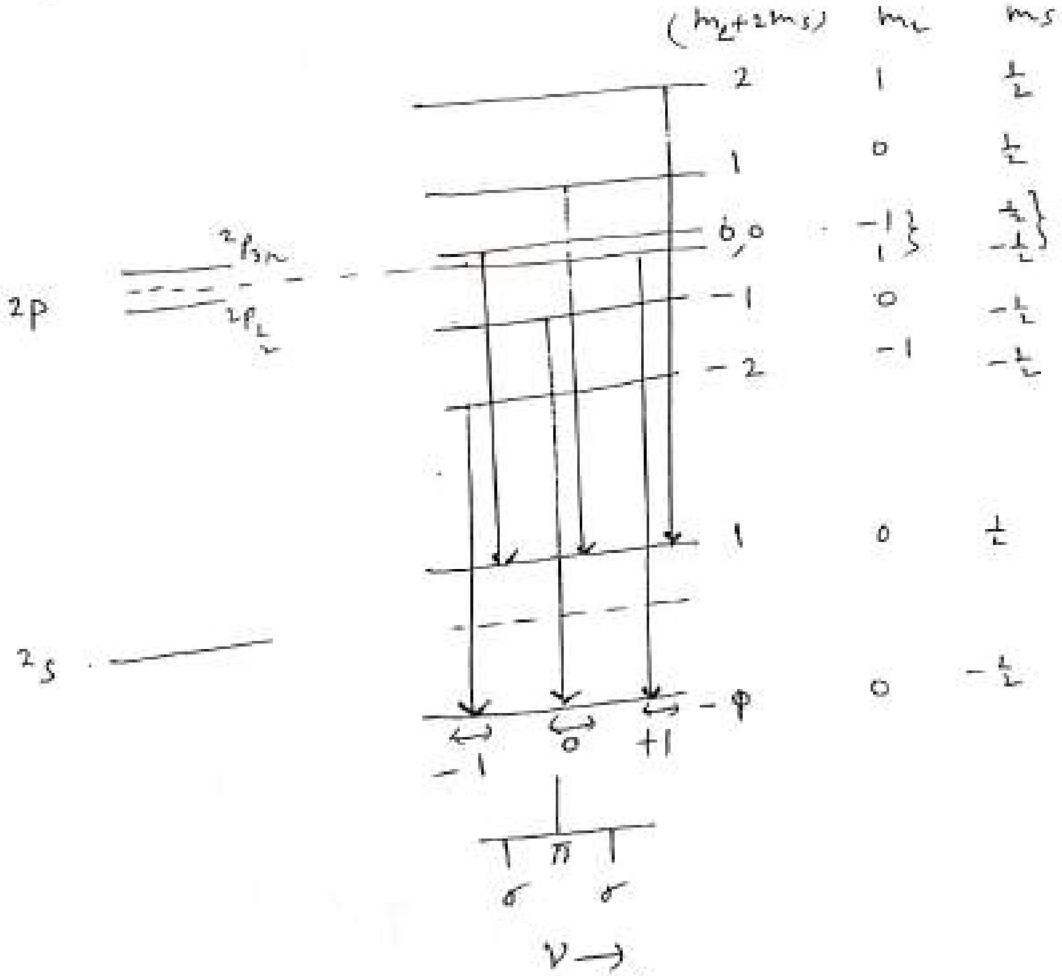
Let us consider the transition

$$^2P \rightarrow {}^2S$$

in a strong magnetic field. This transition is responsible for the D_1 and D_2 lines of sodium ($^2P_{3/2,\frac{1}{2}} \rightarrow {}^2S_{\frac{1}{2}}$). The strong-field levels and the magnetic shifts for the multiplet terms $2p$ and $2s$ are as follows:

Term	No. of strong-field levels $(2L+1)(2S+1)$	m_L	m_s	Shift in Lorentz unit (m_L+2m_s)
$2p$ $L=1, S=\frac{1}{2}$	6	1 0 -1	$\frac{1}{2}, -\frac{1}{2}$ $\frac{1}{2}, -\frac{1}{2}$ $\frac{1}{2}, -\frac{1}{2}$	2, 0 1, -1 0, -2
$2s$ $L=0, S=\frac{1}{2}$	2	0	$\frac{1}{2}, -\frac{1}{2}$	1, -1

The strong-field splittings of the terms $2p$ and $2s$ have been shown in figure.



Strong-field pattern

Example of
Normal Zeeman Effect :-

$^1F_3 - {}^1D_2$: It is a singlet–singlet transition and would give rise to a normal Zeeman triplet.

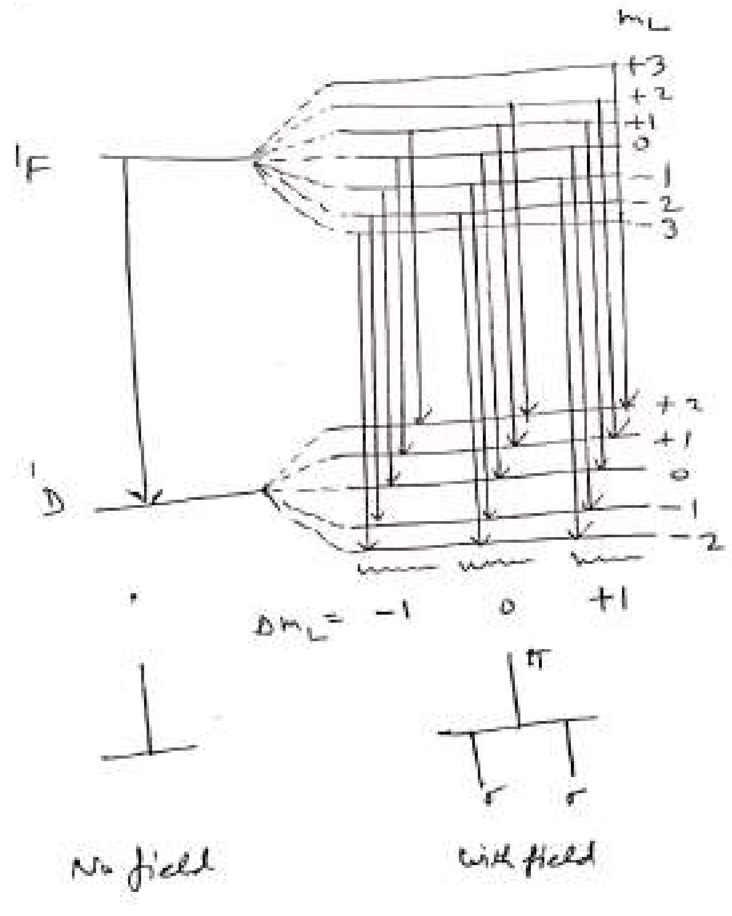
The terms ^1F and ^1D correspond to $L=3$ and $L=2$ respectively and in a weak magnetic field, break into $(2L+1) = 7$ and 5 Zeeman components respectively. The m_L values characterising the Zeeman levels are 3, 2, 1, 0, −1, −2, −3 and 2, 1, 0, −1, −2 respectively.

Since for singlet terms $(S=0, J=L+S=L)$ the Lande 'g' factor is 1, the separation between consecutive Zeeman levels is the same for both terms, equal to one Lorentz unit.

$$\left[g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \right.$$

The Zeeman splitting is different for different J-levels, depending upon the value of g for that level. This also means that the relative separation of the Zeeman levels of one term and those of another are determined by the g-factor alone.

The splitting of the terms is shown in figure



The selection rules $\Delta m_L = 0, \pm 1$ allow 15 transition. Since the Zeeman splitting is same for both terms, all transition corresponding to same Δm_L coincide in wave number. Here we obtain only three Zeeman component lines, the π component corresponding to $\Delta m_L = 0$ and two σ components corresponding to $\Delta m_L = \pm 1$. This is the normal Zeeman triplet.

Example of
Anomalous Zeeman pattern

$^2D_{3/2} - {}^2P_{\frac{1}{2}}$:- It is a doublet-doublet transition so that the Zeeman pattern would be anomalous. The weak-field interaction energy of a one-electron atom is given by

$$-\Delta T = g\, m_J\, L$$

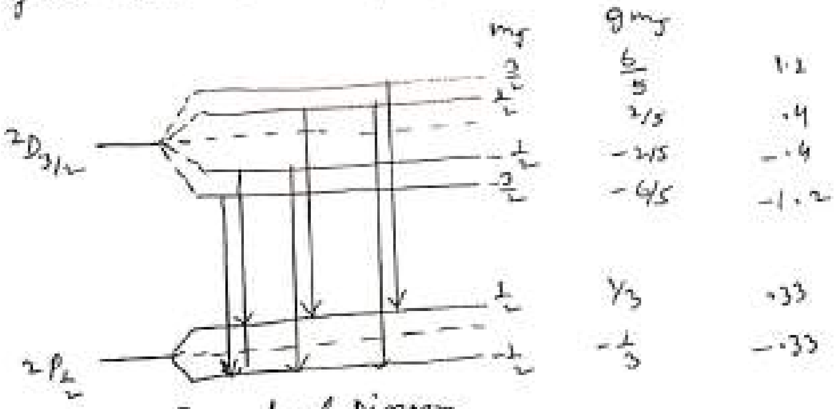
where L is the Lorentz unit. The Lande-g-factor is

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

The Zeeman levels, g-factor and the Zeeman shifts for the given terms $^2P_{\frac{1}{2}}$ and $^2D_{3/2}$ are as follows:

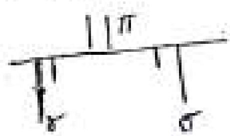
Term	No. of Zeeman levels $(2J+1)$	g	m_J $(+J \ldots -J)$	Zeeman shift in L unit $-\Delta T = g m_J$
$^2P_{\frac{1}{2}}$ $L=1, S=\frac{1}{2}, J=\frac{1}{2}$	2	$\frac{2}{3}$	$\frac{1}{2}, -\frac{1}{2}$	$\frac{1}{3}, -\frac{1}{3}$
$^2D_{\frac{3}{2}}$ $L=2, S=\frac{1}{2}, J=\frac{3}{2}$	4	$\frac{4}{5}$	$\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$	$\frac{6}{5}, \frac{2}{5}, -\frac{2}{5}, -\frac{6}{5}$

The splitting of these terms has been displayed



Energy level Diagram

The Selection rule is $\Delta m_J = 0, \pm 1$



There are in all six allowed transitions, hence six Zeeman components. Transitions corresponding to $\Delta m_J = 0$ give π components polarised parallel to the magnetic field, and transitions corresponding to $\Delta m_J = \pm 1$ give σ components polarised perpendicular to the field, as shown below the energy level diagram.

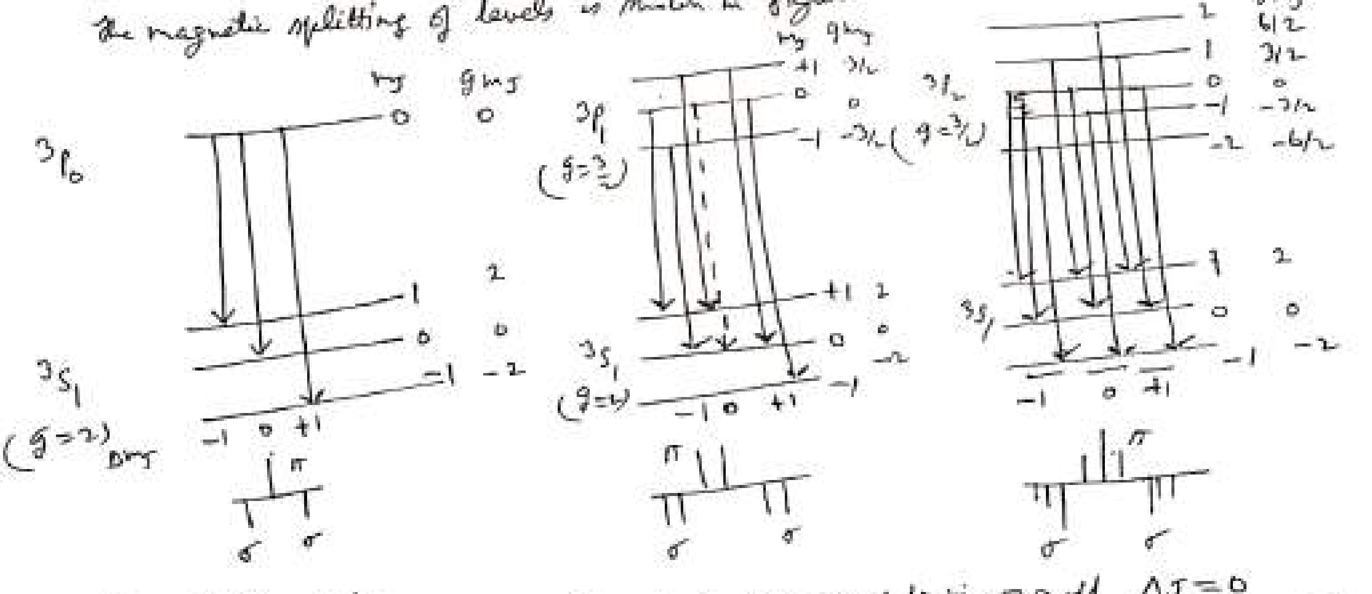
Principal Series Triplet $3P - {}^3S$ or ${}^3P_{0,1,2} - {}^3S_1$:- The fine structure transitions are

${}^3P_0 - {}^3S_1$; ${}^3P_1 - {}^3S_1$; ${}^3P_2 - {}^3S_1$

The g-factor and zeeman shifts for the unperturbed levels ${}^3P_0$, ${}^3P_1$, ${}^3P_2$ and ${}^3S_1$ are as follows :-

unperturbed level	No. of zeeman level $(2J+1)$	g	m_J $(+J \ldots -J)$	Zeeman Shift in Lorentz unit $-\Delta T = g m_J$
${}^3P_0$ $L=1, S=1, J=0$	1	$\frac{0}{0}$	0	0
${}^3P_1$ $L=1, S=1, J=1$	3	$\frac{3}{2}$	$1, 0, -1$	$\frac{3}{2}, 0, -\frac{3}{2}$
${}^3P_2$ $L=1, S=1, J=2$	5	$\frac{3}{2}$	$2, 1, 0, -1, -2$	$\frac{6}{2}, \frac{3}{2}, 0, -\frac{3}{2}, -\frac{6}{2}$
${}^3S_1$ $L=0, S=1, J=1$	3	2	$1, 0, -1$	$2, 0, -2$

The magnetic splitting of levels is shown in figure



The selection rules $\Delta m_J = 0, \pm 1$, but $m_J = 0 \not\leftrightarrow m_J = 0$ if $\Delta J = 0$ allow three zeeman component in the transition ${}^3P_0 - {}^3S_1$, six in ${}^3P_1 - {}^3S_1$ and nine in ${}^3P_2 - {}^3S_1$. The zeeman transition $m_J = 0 \to m_J = 0$ in ${}^3P_1 - {}^3S_1$ is forbidden, since at the same time $\Delta J = 0$ (This transition is indicated by dotted line). The π-components are polarised with electric vector parallel to the magnetic field, the σ-component with electric vector ⊥ to the magnetic field. The sum of the intensities of the π-components is equal to that of the σ-components.

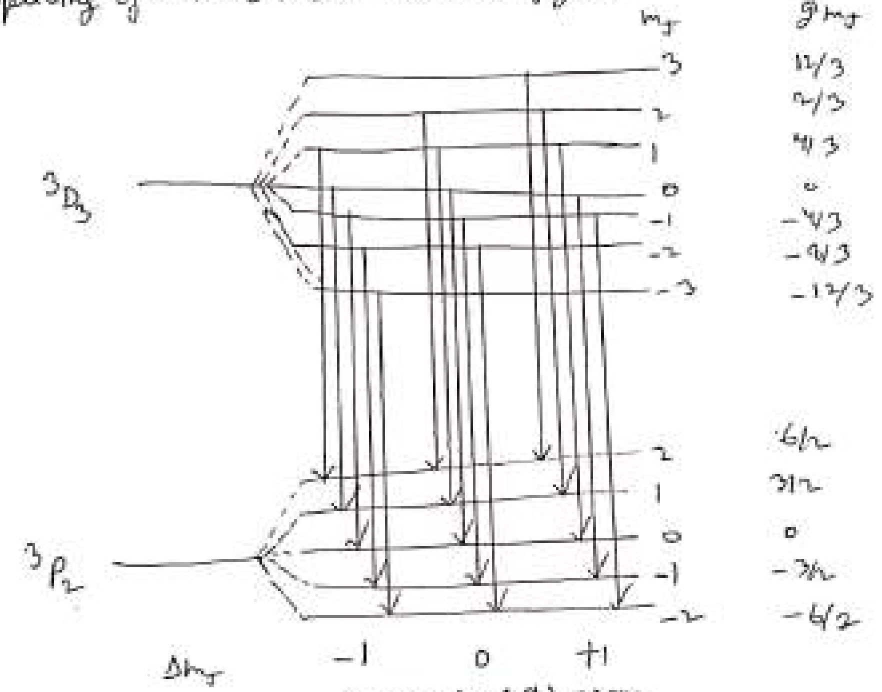
Example of Anomalous Zeeman effect :-

$^3D_3 - {}^3P_2$:- It is a triplet-triplet transition and would give an anomalous Zeeman pattern in a weak external magnetic field. 19

Zeeman levels, g-factors and the shifts from the unperturbed levels $^3D_3$ and $^3P_2$ are as follows :

Unperturbed level	No. of Zeeman levels (2J+1)	g	m_J : J, ..., 0, ..., -J	Zeeman shift in Lorentz Unit $g m_J = -\Delta T$
$^3D_3$ L=2, S=1, J=3	7	$\frac{4}{3}$	3, 2, 1, 0, -1, -2, -3	$\frac{12}{3}, \frac{8}{3}, \frac{4}{3}, 0, -\frac{4}{3}, -\frac{8}{3}, -\frac{12}{3}$
$^3P_2$ L=1, S=1, J=2	5	$\frac{3}{2}$	2, 1, 0, -1, -2	$\frac{6}{2}, \frac{3}{2}, 0, -\frac{3}{2}, -\frac{6}{2}$

The splitting of these levels is shown in figure



Energy level diagram

The selection rules $\Delta m_J = 0, \pm 1$, ($m_J = 0 \nrightarrow m_J = 0$ if $\Delta J = 0$) allow 15 transitions. The rule $\Delta m_J = 0$ give rise to five π components, while $\Delta m_J = \pm 1$ each give five σ components.

The complete pattern is shown below the energy level diagram

Q:- Calculate the wavelength separation between the two component lines which are observed in normal zeeman effect. The magnetic field used is 0.4 weber/m^2, the specific charge $= 1.76 \times 10^{11}$ Coul/kg and $\lambda = 6000$ Å.

Ans:- Wavelength separation between two component lines observed in normal zeeman effect,

$$\Delta\lambda = \frac{eB\lambda^2}{2\pi c m}$$

Specific charge $\frac{e}{m} = 1.76 \times 10^{11}$ Coul/kg

$$\lambda = 6000 \times 10^{-10} m$$

$$B = 0.4 \text{ weber}/m^2$$

$$\Delta\lambda = \frac{0.4 \times 1.76 \times 10^{11} \times (6 \times 10^{-7})^2}{2 \times 3.14 \times 3 \times 10^8}$$

$$= 1.345 \times 10^{11} \text{ metre}$$

$$= 0.1345 \text{ Å}$$

Q:- Find Lande-g factor for $^3P_{\frac{1}{2}}$ energy level.

Ans: Here $L = 1$, multiplicity $2S+1 = 3 \Rightarrow S = 1$, $J = \frac{1}{2}$, $S = 1$

$2S+1 = 3$
$2S = 2$
$S = 1$

For $P \to L = 1$
$S = 3/2, -\frac{1}{2}$
$J = L \pm S$

Lande factor

$$g = 1 + \frac{j(j+1) + S(S+1) - L(L+1)}{2j(j+1)}$$

$$= 1 + \frac{\frac{1}{2}(\frac{1}{2}+1) + 1(1+1) - 1(1+1)}{2 \times \frac{1}{2}(\frac{1}{2}+1)} = 1 + \frac{1}{2} = \frac{3}{2} \text{ Ans}$$

multiplicity $(2S+1)$	P_J → gives value of L for	S = 0, P = 1, D = 2, F = 3

Q:- $^2D_{\frac{3}{2}}$

$2S+1 = 2$
$2S = 1$
$S = \frac{1}{2}$

$J = \frac{3}{2}$

$L = 2$

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Q. Evaluate the Lande's g-factor for (i) pure orbital angular momentum (ii) pure spin angular momentum and (iii) the state $^3P_1$.

Ans:- (i) For pure orbital angular momentum case $S=0$

Hence $J=L$. The Lande's g-factor is then

$$\therefore \quad g = 1 + \frac{L(L+1) - L(L+1)}{2L(L+1)} = 1$$

(ii) For pure spin angular momentum case $L=0$ and thus $J=S$,

$$\therefore \quad g = 1 + \frac{S(S+1) + S(S+1)}{2S(S+1)} = 2$$

(iii) For the state $3P$ we have, $S=1, L=1$ and $J=1$

$$\therefore \quad g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} = 1 + \frac{1(1+1)+1(1+1)-1(1+1)}{2\times1(1+1)} = \frac{3}{2}$$

Q:- Draw the Zeeman pattern for the transition $6\,^1D_2 \to 5\,^1P_1$ of the Cadmium atom.

Ans:- $^1D_2$ implies, $L=2$, $J=2$, $S=0$. In the presence of a magnetic field, this level splits into five levels. On the other hand, $^1P_1$ implies $l=1$, $j=1$ and $S=0$ which splits into 3 levels in a magnetic field. The selection rules to be followed are $\Delta l = \pm1$, $\Delta m_l = 0, \pm1$ and the corresponding energy levels and transitions are illustrated in figure.