Unit-I 6/11

FkeâeF-I

2. (a) State and prove Serret-Frenet formulae.

meejš-øeâveš me$ee keâe eueeKeÙe leLee emeæ keâeepeS~

(b) For a point of the curve

x = c cos hu, y = c sin hu, z = cu

Show that

$$\rho = -\sigma = \frac{2x^2}{c}$$

Where c is constant.

Je›eâ

x = c cos hu, y = c sin hu, z = cu

hej ekeâmee eyevo hej emeæ keâeepeS ekeâ :

$$\rho = -\sigma = \frac{2x^2}{c}$$

pene c Skeâ DeÛej n~

3. (a) Show that osculating plane at point P of the curve has in general three point contact (contact of second order) with the curve at P.

eoKeeFS ekeâ Je›eâ keâ ekeâmee eyevo hej DeeMue<ee meceleue Gme eyevo P hej Je›eâ keâ meeLe meeceevÙele: leeve eyevoÙee mechekeâ (omej Dee[j keâe mechekeâ) jKelee n~

A (Printed Pages 8)

Roll No. ____________

S-686

B.A. / B.Sc. (Part-III) Examination, 2015

(Old Course)

MATHEMATICS - IV

Fourth Paper

(Differential Geometry)

Time Allowed : Three Hours] [***Maximum Marks :*** B.A. : 40, B.Sc. : 75

Note : Attempt five questions in all, selecting one question from each unit. Question No. 1 is compulsory. Symbols have their usual meanings.

øelÙekeâ FkeâeF me Skeâ øeMve Ûevele nS, keâue heeÛe øeMvee keâ Gòej oeepeS~ øeMve me. 1 DeeveJeeÙe n~ øeleekeâe keâ meeceevÙe DeLe n~

1. Attempt all parts : 16/30

(a) Define the curvature and the torsion at any point of a curve. Also define the radius of the curvature and the radius of the torsion at any point of the curve.

Je›eâ keâ ekeâmee eyevo hej Je›eâlee leLee S"velee keâe heejYeeef<ele keâeeepeS~ Je›eâ keâ ekeâmee eyevo hej Je›eâlee $epÙee leLee S"velee e$epÙee keâe Yee heejYeeef<ele keâeeepeS~

(b) Show that curvature K at a point of a curve $\vec{r} = \vec{r}(s)$ is given by

$$K = |\vec{r}' \times \vec{r}''|$$

eoKeeFS ekeâ Skeâ Je›eâ $\vec{r} = \vec{r}(s)$ keâ Skeâ eyevo hej Je›eâlee K evecve me oe peelee n :

$$K = |\vec{r}' \times \vec{r}''|$$

(c) Find first fundamental coefficients E, F, G H for the surface

$$x = u,\ y = v,\ z = u^2 - v^2$$

he‰ $x = u,\ y = v,\ z = u^2 - v^2$ keâ eueS ØeLece DeeOeej ieCeekeâ E, F, G H Øeehle keâeeepeS~

(d) Define first curvature, mean curvature and Gaussian curvature at a point of a surface.

he‰ keâ ekeâmee eyevo hej ØeLece Je›eâlee, ceOÙe Je›eâlee leLee ieeGemeÙeve Je›eâlee keâe heejYeeef<ele keâeeepeS~

(e) Calculate the second fundamental magnitudes of the surface.

$x = a\,(u+v),\ y = b(u - v),\ z = uv$, where a and b are constants.

he‰ $x = a\,(u+v),\ y = b(u - v),\ z = uv$ keâ eÉleeÙe DeeOeej heejceeCee keâe ieCevee keâeeepeS peyeekeâ a, b, eveÙeleeekeâ n~

(f) Define contravarint and covariant vectors.

keââvšeJeejÙevš leLee keâeJeejÙevš meeoMee keâe heejYeeef<ele keâeeepeS~

(g) If A^i be a contravariant vector and B_j is a Covariant vector. Prove that A^iB_j is a tensor of type (1,1).

Ùeeo A^i Skeâ Øeelehee jJelee meeoMe n leLee B_j Skeâ menheejJelee meeoMe n~ emeæ keâeeepeS ekeâ A^iB_j Skeâ (1,1) lejn keâe ØeeoMe n~

(h) Prove that A^iB_i is invariant or scalar.

emeæ keâeeepeS ekeâ A^iB_i Skeâ DeeoMe Ùee DeheejJeleveeÙe n~

(i) Define Ricci tensor. Prove that Ricci tensor is symmetric.

ejkeâe ØeeoMe keâe heejYeeef<ele keâeeepeS~ emeæ keâeeepeS ekeâ ejkeâe ØeeoMe meceecele nelee n~

(j) Define curvature tensor. In a Riemannian manifold, prove that

$$R^i_{jkl} = -R^i_{jkl}$$

Je›eâlee ØeeoMe keâe heejYeeef<ele keâeeepeS~ Skeâ jeceeve yencekeâ ce emeæ keâeeepeS ekeâ

$$R^i_{jkl} = -R^i_{jkl}$$

jeceeve yenceKe ce eoKeeFS ekeâ ceeŠkeâ (ojekeâ) ØeeoMe g_{ij} menheejJelee DeÛej n~

(b) If ϕ is scalar functions of coordinates, show that curl (ϕ grad ϕ) = 0

Ùeeo ϕ eveoMeekeâe keâe DeeoMe Heâueve n lee eoKeeFS ekeâ

curl (ϕ grad ϕ) = 0

9. (a) Prove that in the Riemannian manifold :

(i) $R_{ijkl} = -R_{ijlk}$

(ii) $R_{ijkl} + R_{iklj} + R_{iljk} = 0$

(iii) $R^i_{jkl,m} + R^i_{jlm,k} + R^i_{jmk,l} = 0$

where $R_{ijkl} = g_{ih} R^h_{jkl}$

jeceeve yenceKe ce emeæ keâeepeS ekeâ :

(i) $R_{ijkl} = -R_{ijlk}$

(ii) $R_{ijkl} + R_{iklj} + R_{iljk} = 0$

(iii) $R^i_{jkl,m} + R^i_{jlm,k} + R^i_{jmk,l} = 0$

pene $R_{ijkl} = g_{ih} R^h_{jkl}$

(b) Prove that every Riemannian manifold of constant curvature is an Einstein manifold.

emeæ keâeepeÙe ekeâ ØelÙekeâ DeÛej Je›eâlee Jeeuee jeceeve yenceKe Skeâ DeeFmeševe yenceKe nelee n~

(b) Prove that the necessary and sufficient condition for a curve to be a helix is that its curvature and torsion are in constant ratio.

emeæ keâeepeS ekeâ Skeâ Je›eâ keâe keâC[uee neve keâ eueS DeeJeMÙekeâ leLee GheÙeòeâ Mele Ùen n ekeâ Je›eâ keâe Je›eâlee leLee S"velee keâe Deveheele eveÙeleekeâ nevee ÛeeenS~

Unit-II 6/11

FkeâeF-II

4. (a) If θ is the angle at the point (u,v) between the two directions given by

$$Pdu^2+2Qdu\,dv + Rdv^2 = 0,$$

Show that $\tan\theta = \dfrac{2H\,(Q^2 - PR)^{1/2}}{ER - 2FQ + GP}$

Ùeeo θ ekeâmee eyevo (u,v) hej oe nF eoMeeDee $Pdu^2+2Qdu\,dv + Rdv^2 = 0$ keâ yeeÛe keâe keâeCe n eoKeeFS $\tan\theta = \dfrac{2H\,(Q^2 - PR)^{1/2}}{ER - 2FQ + GP}$.

(b) Prove that, on general surface, a necessary and sufficient condition that the curve v=c be a geodesic is

$$EE_2 + FE_1 - 2EF_1 = 0$$

emeæ keâeepeS ekeâ ekeâmee meeceevÙe he‰ hej Je›eâ v=c keâ DeJeceeleje neve keâe DeeJeMÙekeâ leLee heÙeehle Mele $EE_2 + FE_1 - 2EF_1 = 0$ n~

5\. (a) Find the second order fundamental magnitudes for the surface

$x = u \cos v$, $y = u \sin v$, $z = cv$. Also prove that this surface is minimal.

he‰ $x = u \cos v$, $y = u \sin v$, $z = cv$ keâ eÉleeÙe ceKÙe heejceeCee keâe Øeehle keâeepeS~ Ùen Yee keâeepeS ekeâ Ùen he‰ eceeveceue n~

(b) State and prove Rodrigue formula.

jee[ie me$e keâe GuueKe keâjle nS melÙeehele keâeepeS~

Unit-III 6/11

FkeâeF-III

6\. (a) Obtain the equations of Weingarten.

eJeieeŠve keâ meceekeâjCee keâe Øeehle keâeepeS~

(b) Show that every second order tensor can be expressed as the sum of two tensors, one symmetric and other Skew-symmetric tensor of second order.

eoKeeFS ekeâ ØelÙekeâ eÉleeÙe keâeeŠ keâe ØeeOMe oe ØeeOMee keâ Ùeeie keâ ¤he ce eueKee pee mekeâlee n, epevece Skeâ

eÉleeÙe keâeeŠ keâe meceecele ØeeOMe leLee omeje eÉleeÙe keâeeŠ keâe Øeelemeceecele ØeeOMe neiee~

7\. (a) If T_i be components of a cavariant vector, show that $\left(\frac{\partial T_i}{\partial x^j} - \frac{\partial T_j}{\partial x^i}\right)$ are components of a skew-symmetric covariant tensor of second order.

Ùeeo T_i menheejJelee meeOMe keâ leŠkeâ n~ lee eoKeeFS ekeâ $\left(\frac{\partial T_i}{\partial x^j} - \frac{\partial T_j}{\partial x^i}\right)$ eÉleeÙe keâeeŠ keâe Øeelemeceecele menheejJelee leŠkeâ n~

(b) If $T_{ij} U^i V^j$ is scalar for Contravariant vectors U^i and V^j. Show that T_{ij} are components of a covariant tensor of second order.

Ùeeo Øeelehee jJelee meeOMee U^i SJe V^j keâ eueS $T_{ij} U^i V^j$ Skeâ DeeOMe n~ lee emeæ keâeepeS ekeâ T_{ij} eÉleeÙe keâeeŠ keâ menheejJelee ØeeOMe keâ leŠkeâ n~

Unit-IV 6/12

FkeâeF-IV

8\. (a) In Riemannian manifold, show that metric tensor g_{ij} is covariant constant.