(ii) $\{(1,2),\ (3,4)\}$

(b) Define a quadratic form : Let q be a quadratic form on R^2 defined by :

(i) $q\ (x_1,x_2) = x_1^2 + 9x_2^2 + 3x_1x_2$

(ii) $q\ (x_1,x_2) = x_1^2 + x_1x_2$

Find the symmetric matrix of bilinear form f corresponding to each q.

Skeâ eÉleeleeÙe meceleeele keâe heejYeeef<ele keâeepeS~ ceevee q, R^2 hej eÉleeleeÙe meceleeele n, pee-

(i) $q\ (x_1,x_2) = x_1^2 + 9x_2^2 + 3x_1x_2$

(ii) $q\ (x_1,x_2) = x_1^2 + x_1x_2$

Éeje heejYeeef<ele n~ ØelÙekeâ q keâ meehe#e, eÉ-Skeâleeleeeelee meceleeele f keâ meceecele DeeJÙen %eele keâeepeS~

9. (a) State and prove Cauchy-Schwartz inequality.

keâeMee-MJeepe Demeeecekeâe keâe keâLeve keâeepeS SJe emeæ keâeepeS~

(b) Apply Gram-Schmidt process to the vectors $\alpha_1 = (2,0,1)$; $\alpha_2 = (3,-1,5)$; $\alpha_3 = (0,4,2)$; To obtain an orthonormal basis for R^3 with respect to standard inner product.

«eece-eMceš heæele keâe meeoMee $\alpha_1 = (2,0,1)$; $\alpha_2 = (3,-1,5)$; $\alpha_3 = (0,4,2)$ hej ØeÙeeie keâjkeâ R^3 ce ceevekeâ Deelejie Ceve keâ meehe#e ØemeeceevÙe ueeeyekeâ DeeOeej %eele keâeepeS~

A (Printed Pages 8)

Roll No. ____________

S-678

B.A./B.Sc. (Part-III) Examination, 2015

MATHEMATICS

Second Paper

(Abstract Algebra)

Time Allowed : Three Hours] ***[Maximum Marks :*** B.A. : 35, B.Sc. : 75

Note : Answer five questions in all, selecting one question from each unit. Question No. 1 is compulsory.

ØelÙekeâ FkeâeF me Skeâ ØeMve Ûevele nS, keâue heeÛe ØeMvee keâ Gòej oeepeS~ ØeMve me. 1 DeeveJeeÙe n~

1. Attempt all parts : 15/30

meYee KeC[nue keâeepeS :

(a) Let G be a group of positive real numbers under multiplication. Is the mapping $f : G \rightarrow G$, defined by $f(x) : x^2$, on automorphism of G?

ceevee G Oeveelcekeâ JeemleeJekeâ meKÙeeDee keâe ieCeve keâ meehe#e mecen n~ keÙee ØeeleeÛe$eCe $f : G \rightarrow G$ pee $f(x) : x^2$, Éeje heejYeeef<ele n, G keâe mJekeâeejlee n?

(b) Show that the normalizer N(a) of the element a of a group G, is a subgroup of G.

oMeeFÙe ekeâ mecen G keâ DeJeÙeJe a keâe Øemeeceev Ùekeâ N(a) mecen G keâe Ghemecen n?

(c) State the Eisenstein criterion for the irreducibility of a polynomial with integral coefficients over the rationals. Discuss with an example.

heCeekeâ ieCeekeâe keâ yenheo keâe heejceÙee hej DeKeC[veeÙe neve keâe DeeFmevmeŠeve keâmeeŠe keâe keâLeve keâeepeS~ Goenj Ce meenle mhe° keâeepeS~

(d) Distinguish between a subring and Ideal of a ring.

JeueÙe keâ GheJeueÙe Deej ieCepeeJeuee œ Devlej keâeepeS~

(e) What do you mean by a Simple group? Explain with two examples.

meeOeejCe mecen me Deehekeâe keÙee leelheÙe n? oe GoenjCee meenle JÙeeKÙee keâeepeS~

(f) If F is a field, Prove that its only ideals are (D) and F itself.

Ùeeo F Skeâ #e$e n, lee emeæ keâeepeS ekeâ keâJeue (D) Deej F mJeÙe ne F keâe ieCepeeJeeueÙee n~

(g) Determine whether :

{(1,3,–4), (1, 4, –3) (2, 3, –11)} is a basis of IR^3 or not.

(b) Let $T : R^3 \rightarrow R^3$ be linear operator defined by :

$T(x,y,z) = (3x+z, -2x+y, -x + 2y + 4z)$

Find the matrix of T, with respect to standard basis

ceevee ekeâ $T : R^3 \rightarrow R^3$ Skeâ jeKekeâ mekeâejkeâ n, pee

$T(x,y,z) = (3x+z, -2x+y, -x + 2y + 4z)$

Éeje heejYeeeæle n~ ceevekeâ DeeOeej keâ meehe#e T keâe DeeJÙen %eele keâeepeS~

Unit-IV 5/12

FkeâeF-IV

8. (a) Define a bilinear form. Let f be a bilinear form on R^2 defined by :

$f[(x_1, y_1), (x_2, y_2)] = x_1y_1 + x_2y_2$

Find the matrix of f in each of the following bases :

(i) {(1,0), (0,1)}

(ii) {(1,2), (3,4)}

eÉ-Skeâleeleeùe meceleele keâe heejYeeeæle keâeepeS~ ceevee R^2 hej Skeâ eÉ-Skeâ leeleeÙe meceleele f :

$$f[(x_1, y_1), (x_2, y_2)] = x_1y_1 + x_2y_2$$

Éeje heejYeeeæle n~ f keâe DeeJÙen, evecve DeeOeeje hej %eele keâeepeS -

(i) {(1,0), (0,1)}

n, lee eoKeeFS ekeâ T^{-1}, W me V ce DeeÛÚeokeâ jeKekeâ ¤heevlejCe n~

Unit-I 5/11

FkeâeF-I

2. (a) State and prove Sylow's second theorem.

meeuee keâ eÉleeÙe ØeceÙe keâe keâLeve keâeepeS leLee emeæ keâeepeS~

(b) Let G be a group and $O(G) = p^n$, where p is a prime number. Then show that centre $Z(G) \neq \{e\}$.

ceevee G Skeâ mecen n leLee $O(G) = p^n$ n, pene p Skeâ DeYeepÙe meKÙee n~ leye oMeeFÙe ekeâ keâvõ $Z(G) \neq \{e\}$ n~

3. (a) Prove that the group of inner automorphism of a group G is a normal subgroup of the group of automorphism of G.

emeæ keâeepeS ekeâ mecen G keâe Deevleejkeâ mJeekeâeejlee mecen G keâ mJeekeâeejlee mecen keâe ØemeeceevÙe Ghemecen n~

(b) Prove that the conjugacy relation is an equivalence relation on a group G.

emeæ keâeepeS ekeâ meÙeicee mecyevOe, mecen G hej leuÙelee mecyevOe n~

Unit-II 5/11

FkeâeF-II

4. (a) Prove that an Ideal $A = (a_0)$ is a maximial ideal of the Euclidean ring R if and only if a_0 is a prime element of R.

emeæ keâeepeS ekeâ Skeâ Ùekeueeee[Ùeve JeueÙe R ce ieCepeeJeuee $A = (a_0)$ GeÛÛe‰ n, Ùeeo Deej keâJeue Ùeeo a_0, R keâe DeYeepÙe DeJeÙeJe n~

(b) If R be a Euclidean Ring and $a, b \in R$, If $b \neq 0$, is not a unit in R, then show that

$$d(a) < d(ab)$$

ceevee R Skeâ Ùekeueeee[Ùeve JeueÙe n, Deej $a, b \in R$ n, Ùeeo $b \neq 0$, R ce Øeeleueecee vene n, lee oMeeFÙe ekeâ $d(a) < d(ab)$ n~

5. (a) If f(x) and g(x) are two non-zero elements of F[x] then show that :

Ùeeo f(x) Deej g(x) oe DeMevÙe F[x] keâ DeJeÙeJe n lee emeæ keâeepeS :

$$\deg (f(x).g(x)) = \deg(f(x)) + \deg (g(x)),$$

for $f(x), g(x) \in R[x]$.

(b) State and prove fundamental theorem on homomorphism of rings.

JeueÙe meceekeâeejlee keâe ceue ØeceÙe keâe keâLeve keâeepeS SJe emeæ keâeepeS~

Unit-III 5/11

FkeâeF-III

6. (a) If w be a subspace of finite-dimensional vector space V(F), then show that

$$\dim (v/w) = \dim V - \dim W$$

Ùeeo w heejecele eJeceeÙe meeoMe mecee° V(F) keâ Ghemecee° n lee e

$$\dim (v/w) = \dim V - \dim W$$

(b) Show that the vectors :

$\alpha_1 = (1,1,0,0)$, $\alpha_2 = (0,0,1,1)$

$\alpha_3 = (1,0,0,4)$, $\alpha_4 = (0,0,0,2)$

form a basis of R^4.

oMeeFÙe ekeâ meeoMe :

$\alpha_1 = (1,1,0,0)$, $\alpha_2 = (0,0,1,1)$

$\alpha_3 = (1,0,0,4)$, $\alpha_4 = (0,0,0,2)$

R^4 keâe DeeOeej yeveele n~

7. (a) Let V(F) and W(F) be finite dimensional vector spaces and $T : V \rightarrow W$ be linear transformation. Prove that

rank (T) + nullity (T) = dim V.

ceevee V(F) Deej W(F) heejecele eJeceeÙe meeoMe mecee° Üee n~ $T : V \rightarrow W$ Skeâ jeKekeâ ¤heevlejCe n~ emeæ keâeepeS ekeâ

rank (T) + nullity (T) = dim V

%eele keâeepeS ekeâ :

$\{(1,3,-4), (1, 4, -3)\ (2, 3, -11)\}$ IR^3 keâe DeeOeej n Ùee vene~

(h) Find the characteristic values of the linear operator T on IR^3, defined by

$T(x_1,x_2,x_3) =$

$(3x_1+2x_2+2x_3,\ x_1+2x_2+2x_3,-x_1 - x_2)$

IR^3 hej jeKekeâ mekeâejkeâ T, pee ekeâ

$T(x_1,x_2,x_3) =$

$(3x_1+2x_2+2x_3,\ x_1+2x_2+2x_3,-x_1 - x_2)$

Éeje heejYeee<ele n, keâ DeeYeuee#eeCekeâ ceeve %eele keâeepeS~

(i) Find the symmetric bilinear form corresponding to the quadratic form q on R^2, defined by

$q\ (x_1,\ x_2) = 3x_1x_2 - x_2^{\ 2}$

R^2 hej eÉleelee meceleele q me mecyeevOele eÉSkeâleeleeÙe meceeecele meceleele %eele keâeepeS, pene

$q\ (x_1,\ x_2) = 3x_1x_2 - x_2^{\ 2}$ Éeje heejYeee<ele n~

(j) If a linear transformation $T : V \rightarrow W$ is invertable, then show that T^{-1} is a linear transformation from W on to V.

Ùeeo Skeâ jeKekeâ ¤heevlejCe $T : V \rightarrow W$ JÙeel›eâceCeeÙe