Ùeeo Heâueve f, (a,b) ce Deeyeæ n Deej Fmekeâ Demelelee eyevoDee keâ meceÛÛeÙe keâ meKÙe meeceevle ey n lee eoKeeFÙe ekeâ f, [a,b] ce meceekeâueveeÙe n~

(b) If a function f is continuous in [0,1], then show that

Ùeeo Skeâ Heâueve f, [0,1] ce meelele n lee eoKeeFÙe ekeâ

$$\lim_{n\to\infty} \int_0^1 \frac{n\, f(x)}{1+n^2x^2}\, dx = \frac{\pi}{2}\, f(0)$$

3. (a) If f(x,y) is continuous in $R : a \le x \le b,\ c \le y \le d$ and f(x) is bounded and integrable in (a, b), then $\int_a^b f(x, y)\, f(x)\, dx$ is continuous function of y in [c,d]. Prove it.

Ùeeo f(x,y), $R : a \le x \le b,\ c \le y \le d$ ce meelele n Deej f(x), (a,b) ce Deeyeæ SJe meceekeâueveeÙe n~ lee emeæ keâeepeÙe ekeâ $\int_a^b f(x, y)\, f(x)\, dx$, y keâe [c,d] ce meelelÙe Heâueve n~

(b) Test the convergence of the integral $\int_0^\infty f(x)\, dx$ where f(x) is defined as follows :

meceekeâueve $\int_0^\infty f(x)\, dx$ keâe meelelelee keâe hejeer#eCe keâeepeÙe pene f(x) evecveeekeâle ¤he me heejYeeef<ele n :

A (Printed Pages 8)

Roll No. ____________

S-683

B.A. & B.Sc. (Part-III) Examination, 2015

(Old Course)

MATHEMATICS

First Paper

(Analysis)

Time Allowed : Three Hours] ***[Maximum Marks :*** B.A. : 35, B.Sc. : 75

Note : Attempt five questions in all, choosing one question from each unit. Question No. 1 is compulsory.

ØelÙekeâ FkeâeF me Skeâ ØeMve Ûevele nS, keâue heeÛe ØeMvee keâ Gòej oeepeS~ ØeMve me. 1 DeeveJeeÙe n~ 15/30

1. (a) Show that the constant function k is integrable and $\int_a^b k\, dx = k\,(b-a)$.

eoKeeFÙe ekeâ eveÙele Heâueve k meceekeâueveeÙe n Deej $\int_a^b k\, dx = k\,(b-a)$.

(b) Describe convergence of the improper integral of 2nd kind.

omej Øekeâej keâ FcØeehej meceekeâueve keâe meeleleee JeeCele keâj e~

(c) Show that the integral $\int_0^{\pi/2} x^m \operatorname{cosec}^n x\, dx$ converges if and only if $n < (m+1)$.

eoKeeFÙe ekeâ meceekeâueve $\int_0^{\pi/2} x^m \operatorname{cosec}^n x\, dx$ meelelÙe n Ùeeo Deej keâJeue Ùeeo $n < (m+1)$.

(d) Using Weirestrass's M test show that $\int_0^{\infty} e^{-x^2} \cos yx\, dx$ is uniformly convergent in $(-\infty, \infty)\ \forall y$.

Jeejmešeme keâ M- hej e#eCe keâe ØeÙeeie keâj keâ eoKeeFÙe ekeâ $\int_0^{\infty} e^{-x^2} \cos yx\, dx\ \ \forall y$ Devlejeue $(-\infty, \infty)$ ce Skeâ meceeve ¤he me DeeYemeeje n~

(e) Show that the following function f is differentiable at the origin.

eoKeeFÙe ekeâ evecveekeâle Heâueve f ceue eyevo hej DeJekeâueveeÙe n~

$$f(x, y) = \begin{cases} xy\dfrac{(x^2 - y^2)}{(x^2 + y^2)} & , \text{ if } \quad x^2 + y^2 \neq 0 \\ 0 & , \text{ if } \quad x = y = 0 \end{cases}$$

(f) Find the value of log (1).

log (1) keâe ceeve %eele keâeepeÙe~

(g) Explain Cross Ratio.

›eâeme Deveheele keâe JeeCele keâeepeÙe~

(h) What do you mean by fixed points of a Bilinear Transformations?

eÉjKeeÙe ¤heevlejCe keâ eveÙele eyevoDee keâe mecePeeFÙe~

(i) Give geometrical interpretation of triangular inequality for complex numbers.

meecceßе meKÙeeDee keâ e$eYepeeÙe Demeceeevelee eâe jKeeieeCeleeÙe DeJeOeejCee oeepeÙe~

(j) Show that each closed sphere is a closed set.

eoKeeFÙe ekeâ ØelÙekeâ yevo ieeuee Skeâ yevo meceÛÛeÙe n~

Unit-I 5/11

FkeâeF- I

2. (a) If the function f is bounded in [a,b] and the set of its points of discontinuity has a finite number of limit points then f is integrable in [a,b]. Show it

(b) Let A, B be two subsets of a metric space (x,d). Then Prove that

$Int(A \cap B) = Int\ A \cap Int B.$

Ùeeo A, B ojekeâ mecee° (x,d) keâ GhemeceÛÛeÙe nleè emeæ keâeepeÙe ekeâ $Int(A \cap B) = Int\ A \cap Int B.$

9. (a) State and Prove Cantor's Intersection Theorem.

keâvšj keâ ØeeleÛÚove ØeceÙe keâe keâLeve SJe emeæ keâeepeÙe~

(b) Prove that every complete metric space (x,d) is of second category.

emeæ keâeepeÙe ekeâ ØelÙekeâ mecheCe ojekeâ mecee° (x,d), eÉleeÙe mlej keâe n~

$$f(x) = 1 \quad \text{for} \quad 0 \le x \le 1$$
$$= 0 \quad \text{for } n-1 < x \le n - \frac{1}{n}$$
$$= (-1)^{n+1} \quad \text{for } n - \frac{1}{n} < x \le n$$

where n = 2, 3, 4, 5, ...

Unit-II 5/11

FkeâeF-II

4. (a) If Σu_n is a convergent series of positive terms and a_1, a_2, a_3 .... is a bounded sequence of real numbers, positive or negative, then $\Sigma a_n u_n$ is absolutely convergent. Show it.

Ùeeo Σu_n Oeveelcekeâ heoe keâe Skeâ DeeYemeeje ßeCee n Deej a_1, a_2, a_3 .... Skeâyeæ meekeJeme n~ lee eoKeeFÙe ekeâ $\Sigma a_n u_n$ Skeâ eve<he#e DeeYemeeje ßeCee n~

(b) Show that both the partial derivatives of f(x,y) exist at (0,0) but it is not continuous at (0,0). If

eoKeeFÙe ekeâ (0,0) hej Heâueve f(x,y) keâ oevee DeeeMekeâ DeJekeâueve DeemlelJe ce n ueekeâve Heâueve melele vene n Ùeeo

$$f(x,y) = \begin{cases} xy/(x^2+y^2) & ,(x,y) \ne (0,0) \\ 0 & ,(x,y) = (0,0) \end{cases}$$

5. (a) Find the Fourier series consisting of co-sine terms only. Which represents the periodic function $f(x) = x$ in $0 \le x \le \pi$.

keâepÙee heoe Jeeuee Heâeej Ùej ßeCee %eele keâeepeÙe pee ekeâ $0 \le x \le \pi$ ce DeeJelee Heâueve $f(x) = x$ eve¤ehele keâjlee n~

(b) Find the region into which the line $y = c_2$ $(c_2 \ne 0)$ is mapped the transforma-tion by $w = 1/z$.

Jen #e$e %eele keâeepeÙe #eelepe jKee $y = c_2$ $(c_2 \ne 0)$ keâe eve¤ehele keâjlee n~ epemece ¤heevlejCe $w = 1/z$

Unit-III 5/11

FkeâeF-III

6. (a) Discuss Geometrical representation of z_1/z_2.

z_1/z_2 keâ pÙeeeceleeÙe eve¤heCe keâe eJeJeÛevee keâj~

(b) Given that the function $f(z) = z^2 = x^2 - y^2 + i2xy$ is differentiable everywhere and $f'(z) = 2z$. Verify that the Cauchy-Riemann equations are satisfied everywhere.

eoÙee n ekeâ Heâueve $f(z) = z^2 = x^2 - y^2 + i2xy$ meJe$e

DeJekeâueveeÙe n Deej $f'(z) = 2z$ lee hee° keâeepeÙe ekeâ keâeMee-jeceeve meceekeâjCe meJe$e mele° nelee n~

7. (a) Prove that $u = y^3 - 3x^2y$ is a harmonic function. Determine its harmonic conju-gate and find the corresponding analytic function $f(z)$ in terms of z.

eoKeeFÙe ekeâ $u = y^3 - 3x^2y$ Skeâ ØemeJeeoe Heâueve n~ Fme ØemeJeeoe meÙeicee %eele keâeepeÙe SJe z keâ heoe ce leovemeej JeMuee<ekeâ Heâueve %eele keâeepeÙe~

(b) If $f(z) = u + iv$ is an analytic function then show that u and v are both Harmonic functions.

Ùeeo $f(z) = u + iv$ Skeâ JeMuee<ekeâ Heâueve n lee eoKeeFÙe ekeâ u Deej v oevee ØemeJeeoe Heâueve n~

Unit-IV 5/12

FkeâeF-IV

8. (a) Prove that a set is open if and only if it is union of open spheres.

emeæ keâeepeÙe ekeâ Skeâ meceÛÛeÙe Keuee neiee Ùeeo Deej keâJeue Ùeeo Ùen Keue ieeuee keâe mece—