(b) Solve nue keâeepeS :

$$y'' + 3y' - 10y = 6e^{4x}$$

3. (a) Solve the following equation by the method of variation of parameters:

evecve meceekeâjCe keâe nue ØeeÛeue eJeÛejCe eJeeOe Éeje %eele keâeepeS :

$$(x^2-1)\, y'' - 2xy' + 2y = (x^2-1)^2$$

(b) Solve nue keâeepeS :

$$y'' - 2y' = 12x - 10 .$$

Unit-II / FkeâeF-II 4/7½

4. (a) Find power Series solution of equation :

$$y'' + y' - xy = 0$$

meceekeâjCe $y'' + y' - xy = 0$ keâe leele ßeCee keâ ®he ce nue %eele keâeepeS~

(b) Prove that emeæ keâeepeS :

$$J_P(-x) = (-1)^P J_P(x)$$

5. (a) Prove that emeæ keâeepeS :

$$P_n(x) = \frac{1}{2^n \lfloor n} \cdot \frac{d^n}{dx^n}(x^2-1)^n$$

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B.A./B.Sc. (Part-II) Examination, 2015

MATHEMATICS

Third Paper

(Differential Equations)

Time Allowed : Three Hours] ***[Maximum Marks :*** B.A. : 25, B.Sc. : 50

Note : Answer five questions in all, choosing one question from each unit. Question No.1 is compulsory. Symbols have their usual meanings.

ØelÙekeâ FkeâeF me Skeâ ØeMve Ûevele nS, keâue heeÛe ØeMvee keâe nue keâeepeS~ ØeMve me.1 DeeveJeeÙe n~ Øeleekeâe keâ meeceevÙe DeLe n~

1. (a) Find particular solution of $y''-y'-6y=e^{-x}$.

meceekeâjCe $y''-y'-6y=e^{-x}$ keâe eJeeMe$e %eele keâeepeS~ 10/20

(b) Verify that $y_1=x$ is one solution of $y''-xf(x)y'+f(x)y=0$, and find general solution.

melÙeeheле keâeepeS ekeâ $y_1 = x$ meceekeâjCe

$y''-xf(x)y'+f(x)y=0$ keâe Skeâ nue neie, Deej heCe nue %eele keâeepeS~

(c) Determine the nature of point $x=0$, for the equation, $x^3y''+(\cos 2x-1)y'+2xy=0$.

meceekeâjCe $x^3y''+(\cos 2x-1)y'+2xy=0$ keâe eueS eyevo $x=0$ keâe mJe®he %eele keâeepeS~

(d) Show that $J_0^1(x) = -J_1(x)$.

oMeeFS ekeâ $J_0^1(x) = -J_1(x)$.

(e) Prove that (emeæ keâeepeS) :

$$P_n^1(1) = \frac{1}{2} n\,(n+1)$$

(f) Prove that (emeæ keâeepeS) :

$$2F_1\ (a,\ b,\ b\ :\ x)\ =\ (1-x)^{-a}$$

(g) Define radius of convergence of power series.

leele ßeCee keâe DeeYemejCe e$epÙee keâe heejYee<ee oeepeS~

(h) Define orthogonal and orthonormal set of functions on interval [a,b].

Delejeue [a,b] hej Heâuevee keâe ueeeyekeâ SJe ØemeeceevÙe ueeeyekeâ meceÛÛeÙe keâe heejYee<ele keâeepeS~

(i) Find the general solution of the system :

evekeâeÙe keâe nue %eele keâeepeS :

$$\begin{cases} \dfrac{dx}{dt} = x \\ \dfrac{dy}{dt} = y \end{cases}$$

(j) Find critical points and differential equation of path of the system :

evekeâeÙe : $\dfrac{dx}{dt} = 2x^2y$

$$\frac{dy}{dt} = x\,(y^2-1)$$

keâ >eâeelekeâ eyevo SJe Je>eâ keâe DeJekeâue meceekeâjCe %eele keâeepeS~

Unit-I / FkeâeF-I 4/7½

2. (a) Prove that if $y_1(x)$ and $y_2(x)$ are any two solutions of equation; $y''+P(x)y'+Q(x)y=0$ then their Wronskian is either identically zero of never zero on [a b].

emeæ keâeepeS ekeâ Ùeeo $y_1(x)$ Deej $y_2(x)$ meceekeâjCe $y''+P(x)y'+Q(x)y=0$ keâ oe nue n lee Gvekeâe jemeekeâÙeve Ùee lee MevÙe neiee Ùee keâYee MevÙe vene neiee~

tem :

euеÙeehÙeveeJe keâe ØeIÙe#e eJeeOe Éeje evecve keâeÙe keâ ›eâeIeeÙe eyevo (0,0) keâ mLeeeÙeIJe keâe hejeʼneCe keâeepeS :

$$\frac{dx}{dt} = -3x^3 - y$$

$$\frac{dy}{dt} = x^5 - 2y^3$$

(b) Prove that emeæ keâeepeS ekeâ :

$$2\,F_1(1, 1\,;\,2, -x) = \sum_{n=0}^{\infty} \frac{(-x)^n}{n+1} \text{ for } |x| < 1$$

Unit-III / FkeâeF-III 4/7½

6. (a) Find all eigen values and eigen functions of Sturm-Liouville Problem :

mšce-uÙeeJeue mecemÙee :

$$y'' + \lambda y = 0,\; y(0) = 0,\; y'\left(\frac{\pi}{2}\right) = 0$$

keâ meYee DeeYeuee#eeCekeâ ceeve IeLee DeeYeuee#eeCekeâ Heâueve %eeIe keâeepeS~

(b) Prove that emeæ keâeepeS :

$$\int_{-1}^{1} P_n(x)\, P_m(x)\, dx = 0, \quad m \neq n$$

7. (a) Prove that the function $1-x$ and $1-2x+\frac{x^2}{2}$ are orthogonal with respect to weight function e^{-x} on $(0, \infty)$.

emeæ keâeepeS ekeâ Heâueve $1-x$ SJe $1-2x+\frac{x^2}{2}$, DeIejeue $(0, \infty)$ Yeej Heâueve e^{-x} keâ meehe#e ueeeyekeâ n~

(b) Show that the set $\{1, \cos 2x, \cos 4x, \cos 6x, \ldots\}$

is orthogonal set of function on an interval $[0, \pi]$ and determine the orthonormal set.

oMeeFS ekeâ meceÛÛeÙe $\{1, \cos 2x, \cos 4x, \cos 6x, \ldots\}$ Dele jeue $[0, \pi]$ hej ueeeyekeâ Heâuevee keâe meceÛÛeÙe n leLee Øemeeceevùe meceÛÛeÙe Yee Øeehle keâeepeS~

Unit-IV / FkeâeF-IV 4/7½

8. (a) Solve nue keâeepeS :

$$\frac{dx}{dt} = -3x + 4y$$

$$\frac{dy}{dt} = -2x + 3y$$

(b) If $x = x_1(t)$, $y = y_1(t)$, and $x = x_2(t)$, $y = y_2(t)$, are two solutions of the system :

$$\frac{dx}{dt} = a_1(t)x + b_1(t)y$$

$$\frac{dy}{dt} = a_2(t)x + b_2(t)y$$

an [a, b], then prove that they are linearly dependent on this interval if and only if thier Wronskian is identically zero.

emeæ keâeepeS ekeâ evekeâeÙe

$$\frac{dx}{dt} = a_1(t)x + b_1(t)y$$

$$\frac{dy}{dt} = a_2(t)x + b_2(t)y$$

keâ oe nue $x = x_1(t)$, $y = y_1(t)$, Deej $x = x_2(t)$, $y = y_2(t)$, jeKekeâle: Deeeßele neie, Fmekeâ eueS Ùen DeeJeMÙekeâ n ekeâ Gvekeâe jemeekeâÙeve MevÙe ne, meeLe ne emeæ keâeepeS ekeâ Ùen heÙeehle Mele Yee n~

9. (a) Find the nature of critical point, sketch the phase portrait and discuss the stability of the critical point of the system :

evecve evekeâeÙe keâ ›eâeleeÙe eyevo keâe Øekeâele %eele keâeepeS, evekeâeÙe keâe Heâpe hee jšš ®ehele keâeepeS leLee ›eâeleeÙe eyevo keâ mLeeeÙelJe hej eJeJeÛevee keâeepeS :

$$\frac{dx}{dt} = 4x - 2y$$

$$\frac{dy}{dt} = 5x + 2y$$

(b) Examine the stability of the critical point (0,0) by Liapunov's direct method of sys-