meceekeâ jCe

$2x^2-7y^2+2z^2-10yz-8zx-10xy+6x+12y$
$-6z+5=0$ keâe ceevekeâ ¤he ce yeoeueS~

Unit - IV 3/7½

FkeâeF- IV

8. (a) Prove that

emeæ keâeepeS ekeâ

$$i\log\left(\frac{x-i}{x+i}\right)=\pi-2\tan^{-1}x$$

(b) Prove that

emeæ keâeepeS ekeâ

$$\log\tan\left(\frac{\pi}{4}+\frac{ix}{2}\right)=i\tan^{-1}(\sinh x)$$

9. (a) Sum the series

$$1-\frac{c^2\cos 2\theta}{\lfloor 2}+\frac{c^4\cos 4\theta}{\lfloor 4}-\frac{c^6\cos 6\theta}{\lfloor 6}+\ldots\ldots\infty$$

ßeCee

$$1-\frac{c^2\cos 2\theta}{\lfloor 2}+\frac{c^4\cos 4\theta}{\lfloor 4}-\frac{c^6\cos 6\theta}{\lfloor 6}+\ldots\ldots\infty$$

keâe Ùeeie %eele keâeepeS~

(b) Prove that

emeæ keâeepeS ekeâ

$$\frac{\pi}{4}=\left(\frac{2}{3}+\frac{1}{7}\right)-\frac{1}{3}\left(\frac{2}{3^3}+\frac{1}{7^3}\right)+\frac{1}{5}\left(\frac{2}{3^5}+\frac{1}{7^5}\right)\ldots\ldots\infty$$

A (Printed Pages 8)

Roll No. ____________

S-672

B.A./B.Sc. (Part-I) Examination, 2015

(Regular)

MATHEMATICS

Fourth Paper

(Geometry & Trigonometry)

Time Allowed : Three Hours] ***[Maximum Marks :*** { B.A. : 25, B.Sc. : 50 }

Note : Attempt five questions in all, choosing one question from each unit. Question No. 1 is compulsory.

ØelÙekeâ FkeâeF me Skeâ ØeMve uekeâj Ûevele nS, keâue heeÛe ØeMvee keâe nue keâeepeS~ ØeMve meKÙee 1 DeeveJeeÙe n~

1. Attempt all parts : 10/20

meYee Yeeie nue keâeepeS :

(i) Prove that the line $\frac{l}{r}=A\cos\theta+B\sin\theta$ is tangent to the conic $\frac{l}{r}=1+e\cos\theta$ if $(A-e)^2+B^2=1$.

emeæ keâeepeS ekeâ jKee $\frac{l}{r}=A\cos\theta+B\sin\theta$ MeekeâJe $\frac{l}{r}=1+e\cos\theta$ keâe mheMe keâjlee n, Ùeeo

$(A-e)^2+B^2=1$

(ii) Prove that all conics through the intersections of two rectangular hyperbolas are themselves rectangular hyperbolas.

emeæ keâeepeS ekeâ oe mecekeâeCeeÙe DeeflehejJeueÙe keâ ØeeflÛÚo eyevo me peeve Jeeue MeekeâJe mJeÙe Skeâ mecekeâeCeeÙe DeeflehejJeueÙe nele n~

(iii) Find the equation of confocals to an ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

MeekeâJee keâe meceekeâjCe %eele keâeepeS pee oeleJeòe

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$ keâ meveeeYe n~

(iv) Find the equation of the plane passing through the point (1,1,–1) and perpendicular to the planes $x+2y+3z-7=0$ and $2x-3y+4z = 0$.

oe meceleuee $x+2y+3z-7=0$ leLee $2x-3y+4z = 0$ keâ uecyeJele leLee eyevo (1,1,–1) me nekeâj iepejve Jeeue meceleue keâe meceekeâjCe %eele keâeepeS~

(v) Find the equation of the sphere whose centre is (2, –3, 4) and passes through the point (1,2, 3).

Gme ieeue keâe meceekeâjCe %eele keâeepeS epemekeâe keâvõ (2, –3, 4) n leLee eyevo (1,2, 3) me nekeâj iepejlee n~

pass through the line $7x+10y=30$, $5y-3z=0$.

MeekeâJepe $7x^2+5y^2+3z^2=60$ keâ Gve mheMe meceleuee keâe meceekeâjCe evekeâeeueS pee jKee $7x+10y=30,\ 5y-3z=0$ me nekeâj peele n~

(b) If the plane $lx+my+nz=p$ passes through the extremities of three conjugate semi-diameters of an ellipsoid :

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

then prove that $a^2l^2+b^2m^2+c^2n^2=3p^2$.

Ùeeo meceleue $lx+my+nz=p$

oeleJeòepe $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ keâ leeve DeæmeÙeicee JÙeemee keâ emeje me nekeâj peelee n, lee emeæ keâeepeS ekeâ $a^2l^2+b^2m^2+c^2n^2=3p^2$.

7. (a) Find the coordinates of centre of the section of the ellipsoid $3x^2+3y^2+6z^2=10$ by the plane $x+y+z=1$.

meceleue $x+y+z=1$ Éeje oeleJeòepe $3x^2+3y^2+6z^2=10$ keâ ekeâS ieS ØeeflÛÚo keâ keâvõ keâ eveoMeekeâ %eele keâeepeS~

(b) Reduce the equation

$2x^2-7y^2+2z^2-10yz-8zx-10xy+6x+12y-6z+5=0$ to the standard form.

Unit - I 4/7½

FkeâeF - I

2. (a) Prove that the locus of the poles of normal chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is the curve

$$\frac{a^6}{x^2}+\frac{b^6}{y^2}=(a^2-b^2)^2$$

emeæ keâeepeS ekeâ oeleJeòe $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ keâ DeeYeuecye peeJeeDee keâ OeJee keâe eyevoheLe Je›eâ

$$\frac{a^6}{x^2}+\frac{b^6}{y^2}=(a^2-b^2)^2$$ n~

(b) Find the condition that two diameters of the conic $ax^2+2hxy+by^2+2gx+2fy+c=0$ which are parallel to the lines $y=mx$ and $y=m'x$ may be conjugate diameters of the conic.

MeekeâJe $ax^2+2hxy+by^2+2gx+2fy+c=0$ keâ oe JÜeemee keâ meÙeicee neve keâe Øeeleyevo %eele keâeepeS pee ekeâ jKeeDee $y=mx$ Deej $y=m'x$ keâ meceeveelej n~

3. (a) Trace the conic

$16x^2-24xy+9y^2-104x-172y+44=0$

MeekeâJe

$16x^2-24xy+9y^2-104x-172y+44=0$

keâe DeeYejKeCe keâeepeS~

(b) In any conic, prove that the sum of the reciprocals of two perpendicular focal chords is constant.

ekeâmee MeekeâJe keâ eueS emeæ keâeepeS ekeâ mecekeâeCeeÙe veeeYeÙe peeJeeDee keâ JÜel›eâce keâe Ùeeie DeÛej nelee n~

Unit - II 4/7½

FkeâeF - II

4. (a) Find the length and equation of the shortest distance between the lines :

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \text{ and } \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$$

oe jKeeDee $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$

leLee $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$

keâ yeeÛe vÙevelece oje leLee vÙevelece oje jKee keâe mecæekeâjCe Øeehle keâeepeS~

(b) Prove that the circle

$x^2+y^2+z^2-2x+3y+4z-5=0$,

$5y+6z+1=0$;

and

$x^2+y^2+z^2-3x-4y+5z-6=0$,

$x+2y-7z=0$

lie on the same sphere and find its equation.

emeæ keâeepeS ekeâ Jeòe

$x^2+y^2+z^2-2x+3y+4z-5=0$,

$5y+6z+1=0$;

leLee

$x^2+y^2+z^2-3x-4y+5z-6=0$,

$x+2y-7z=0$

Skeâ ne ieeue hej emLele n~ Fme ieeue keâe meceeskeâjCe Yee Øeehle keâeepeS~

5. (a) Find the equation of the cylinder whose generators are parallel to the line $z=3x$; $3y+2z=0$ and whose guiding curve is the ellipse $x^2+2y^2=1$; $z=3$.

Gme yeueve keâe meceeskeâjCe %eele keâeepeS epemekeâe pevekeâ jKeeS jKee $z=3x$; $3y+2z=0$ keâ meceevlej n leLee Gmekeâe eveoMekeâ Je›eâ oeleJeòe $x^2+2y^2=1$; $z=3$ n~

(b) Find the equation of right circular cone whose vertex is (3,2,1), axis is the line

$$\frac{x-3}{4}=\frac{y-2}{1}=\frac{z-1}{3}$$

and its semivertical angle is 30^0.

Gme uecyeJeòeeÙe Mekeâ keâe meceeskeâjCe Øeehle keâeepeS epemekeâe Meee<e (3,2,1) n, De#e jKee $\frac{x-3}{4}=\frac{y-2}{1}=\frac{z-1}{3}$ n leLee Gmekeâe DeOe-TOJe keâeCe 30^0 n~

Unit - III 4/7½

FkeâeF - III

6. (a) Find the equation of the tangent planes to the conicoid $7x^2+5y^2+3z^2=60$ which

(vi) Prove that no two generators of the same system intersect.

emeæ keâeepeS ekeâ Skeâ evekeâeÙe keâ oe pevekeâ hejmhej ØeeleÛÚo vene keâjle n~

(vii) Show that the plane $8x-6y-z=5$ touches the paraboloid $\frac{x^2}{2}-\frac{y^2}{3}=z$ and find the coordinates of the point of contact.

eoKeeFÙe ekeâ meceleue $8x-6y-z=5$ hejJeueÙepe $\frac{x^2}{2}-\frac{y^2}{3}=z$ keâe mheMe keâjlee n leLee mheMe eyevog keâ eveoMeekeâ %eele keâeepeS~

(viii) Find the real circular sections of the paraboloid. $x^2+10z^2=2y$

hejJeueÙepe $x^2+10z^2=2y$ keâe JeemleeJekeâ JeòeeÙe heejÛÚo %eele keâeepeS~

(ix) Prove that

$\sinh^{-1}(\cot x) = \log(\cot x + \operatorname{cosec} x)$

emeæ keâeepeS ekeâ

$\sinh^{-1}(\cot x) = \log(\cot x + \operatorname{cosec} x)$

(x) Prove that e^z is a periodic function of period $2\pi i$.

emeæ keâeepeS ekeâ e^z Skeâ DeeJelee Heâueve n epemekeâe DeeJeleveekeâ $2\pi i$ n~